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A RISK-BASED APPROACH TO MODELING LIFE-CYCLE COSTS ASSOCIATED WITH WARRANTY SPECIFICATIONS FOR TRANSPORTATION INFRASTRUCTURE

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A RISK-BASED APPROACH TO MODELING LIFE-CYCLE COSTS ASSOCIATED WITH WARRANTY SPECIFICATIONS FOR TRANSPORTATION INFRASTRUCTURE

by

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Dissertation

Presented to the Faculty of the Graduate School of

the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

May, 2006

ACKNOWLEDGMENTS

I would like to express my gratitude to Dr Zhanmin Zhang, my supervisor, for his invaluable advice, guidance and encouragement. It has been a privilege and pleasure to work with him. I extend my gratitude to all members of my dissertation committee for their constructive comments and suggestions during the course of my study.

I am thankful to all my friends at UT and abroad for their support throughout my studies. At last, but not at least, I am grateful to my father, mother and brother, who have been constantly supportive of my academic pursuit. This wouldn't have been possible without their support.

A RISK-BASED APPROACH TO MODELING LIFE-CYCLE COSTS ASSOCIATED WITH WARRANTY SPECIFICATIONS FOR TRANSPORTATION INFRASTRUCTURE

Publication No._____________

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The University of Texas at Austin, 2006

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To improve quality and reduce the overall costs, in recent years, many state highway agencies have started investigating innovative contracting methods, such as performance-based warranty contracting. This contracting method is structured to shift the performance-related risk from the agency to the contractor, by means of warranty provision. Even though the application of performance-based warranty contracting methods allows for a "win-win" situation, where agencies hedge the performance-related risk, and contractors have more flexibility in the design and construction processes, there are many concerns with its implementation. One of the most important concerns is how to quantify the risk cost.

This dissertation is focused on the development of a robust and flexible methodological framework for quantifying the risk cost associated with warranty specifications for transportation infrastructure. The key components of this framework

for studying performance warranties include: characterization of the warranty systems, development of probabilistic performance models based on the method of moments, formulation of the risk cost quantification models, and formulation of the models for determining the optimal design strategy and maintenance schedule.

In this dissertation, three types of warranty systems are characterized and elaborated upon in detail: short-term, long-term, and maintenance performance warranties. To test the accuracy of the method of moments for developing reliability functions, the current AASHTO method for design of pavements is employed to provide a case study. The results from the comparison analysis of the methods of moments with Monte Carlo simulation indicate that the method of moments yields accurate predictions of the failure probabilities; in general, the quality of estimation improves as the order of the central moments in reliability indices increases. Finally, the methodology is illustrated with numerical examples to show that models for quantifying the risk cost associated with warranty specifications can be developed.

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CHAPTER 1 INTRODUCTION

The selection of an appropriate project delivery method is essential to the success of a project. A project delivery method is the process in which components of procurement, such as planning, design, and construction are combined to complete the project. Under this larger scope, a particular contracting technique used for bidding, managing, and specifying the project deliverables is referred to as a contracting method.

The most commonly used contracting method for procurement and management of transportation infrastructure is the design-bid-build method. Once the design is approved by the agency, the project proceeds to the bidding phase, where the project is awarded to the most qualified bidder. Although this contracting method, in theory, allows for a significant reduction in costs on the front end, it often leads to much higher overall costs, due to the problems associated with enforcing quality control.

To improve quality, promote accountability, and reduce the overall costs, in recent years, many highway and turnpike agencies have started investigating the applicability of innovative contracting methods, in particular the applicability of performance warranty contracting. A distinctive feature of the contracting method with performance warranties is its ability to transfer performance-related risks from the agencies to the contractors. Clearly, in such contracting settings, identifying, quantifying, and managing the performance-related risk is critical.

This chapter introduces the motivation for this work, presents the goals and objectives, summarizes the contributions, and outlines the remainder of this dissertation.

1.1 Background and Motivation

Product warranties are not a new field of study. Over the years, researchers have extensively investigated the impact of different types of warranties on product development. In spite of the success of product warranties, contractors in the highway sector are normally not required to provide a warranty.

In contrast to the traditional design-bid-build contracting approach, warranty contracting requires the contractors to provide a contractual agreement that the constructed facility should not fail under the defined failure criteria for the specified warranty period and/or the level of accumulated traffic applications; if the facility does fail, the contractor is responsible for taking remedial action as specified in the warranty policy. Typically, a failure criterion is defined as the minimal acceptable level of some condition indicator. For pavement structures, the condition indicators can be roughness, rutting, cracking, or a single combined indictor, such as the present serviceability index (PSI). To ensure that the contractor will honor the warranty, the contractor is required to provide a performance bond. This type of warranty is referred to as a performance warranty.

Even though the extent of utilizing performance warranties in the highway sector varies from country to country, there is no doubt that performance warranties are becoming a more common type of contracting method for both new construction and maintenance projects. In the United Kingdom and Spain, under design-build-finance-operate concession agreements, the coverage of performance warranties ranges from 25 to 30 years. In Germany, the coverage of performance warranties is 20 years for maintenance projects, whereas in Denmark the length of maintenance performance warranties varies from 11 to 18 years. Since 1987, when the first warranty contract was implemented in the United States, more than 250 projects based on warranty contracting methods have been completed (Ozbek, 2004). However, the majority of these projects were still simple warranties for materials and workmanship.

An example of the application of performance warranties in the U.S. highway sector is the Corridor 44 project. For a one-time cost of $62 million, the New Mexico State Highway and Transportation Department (NMSHTD) negotiated a performance warranty contract with Koch Industries to design the pavement, manage the construction, and to guarantee the overall pavement performance of a 121-mile long section of Corridor 44 for twenty years. The performance warranty was secured by a $114 million surety bond. The NMSHTD reported that the state would save $89 million in the maintenance costs over the twenty-year period.

Even though performance warranties allow for a “win-win” situation, where agencies hedge the performance risk and contractors have more flexibility in design and construction, there are many concerns with their implementation. In addition to legal issues, estimating the cost of a performance warranty is undoubtedly a major concern. Frequently, the owners and the contractors have different perceptions of the actual amount of this premium cost. The owners are concerned that they are paying for something contractors should already be responsible for, whereas the contractors are aware that by signing a warranty contract, they are faced with a substantial risk that needs to be managed, starting from the design and construction phases, through the phase of maintenance scheduling.

1.2 Research Goal and Objectives

The overall goal of this research is to develop a comprehensive methodological framework to quantify and analyze life-cycle costs associated with performance warranties. The developed framework should be generic and flexible enough to accommodate both the agencies' and the contractors' interests, different warranty policies, and various design approaches. The objectives pertaining to this goal are as follows:

1. Formulate a framework to ***characterize performance warranty systems*** for transportation infrastructure. The developed framework should include all factors influencing the warranty system, cover the whole warranty life-cycle, and be consistent with engineering principles and practices;
2. ***Develop a probabilistic performance model*** that can be used to predict the performance of transportation infrastructure facilities during the warranty period. The developed model should be able to accommodate various warranty specifications and different design approaches. In addition, the performance model should be able to take into account the effects of preventive maintenance and rehabilitation actions;
3. Develop a model that can be used to ***quantify the risk cost*** associated with performance warranties. The model should be able to quantify the risk cost for different warranty specifications. In particular, three types of performance

warranties need to be examined closely: short-term, long-term, and maintenance performance warranties; and

4. Formulate optimization models to ***determine the optimal design strategy and maintenance schedule*** for transportation infrastructure facilities using performance warranties. The optimization models should be formulated in a manner that would allow the implementation of standard optimization algorithms.

1.3 Research Contributions

In a broad sense, this research contributes to the field of transportation infrastructure engineering in two major areas. The first area is the cost estimation for construction and maintenance projects acquired with performance warranties. The developed risk cost model will benefit both the agencies and the contractors. The agencies will benefit from having a measure of the amount of risk that is being transferred from the agency to the contractors, while the contractors will benefit from having a risk measure that could be used to estimate the total lump-sum costs for performance warranty projects. The second area of significance is the application of the method of moments to develop reliability functions. The traditional probabilistic performance models in transportation engineering are developed for specific data sets, failure criteria, and design approaches. The reliability modeling approach in this dissertation is able to encompass different failure criteria and design methods and include the effects of preventive maintenance and rehabilitation actions. Some of the specific contributions in these two major areas include:

1. Development of a comprehensive methodological framework for modeling life-cycle costs for projects contracted with performance warranties;
2. Development of a reliability model that is based on the method of moments, a technique that gives more accurate estimation of failure probability. The model's accuracy is validated with Monte-Carlo simulation;
3. Development of risk cost models for performance warranties. The developed models are sensitive to different discount rates as well as different types of performance warranty specifications;
4. Development of a model that can be used to quantify the effects of preventive maintenance and rehabilitation actions and emergency repairs on the risk cost; and
5. Formulation of optimal warranty-based design and maintenance schedule problems for performance warranties. The optimal design problem includes determining the optimal initial design and corresponding optimal rehabilitation schedule.

1.4 Dissertation Outline

This dissertation is organized in nine chapters. Following this chapter, in which the motivation, objectives, and contributions of this research are introduced, the next chapter presents an overview of the background literature, covering four related topics: general warranty system analysis, probabilistic performance modeling for transportation infrastructure, modeling the effects of preventive maintenance and rehabilitations, and optimization models for design and maintenance.

In Chapter 3, the methodological framework for warranty analysis is presented. The discussion includes characterization of the warranty system, mathematical representation of the system, and the methodological approach for conducting the research. The warranty systems for short-term, long-term, and maintenance performance warranties are also defined and examined in this chapter.

The reliability model based on the method of moments is presented in Chapter 4. This approach is based on the estimates of the first four central moments of the limit state function to determine time-dependent failure probabilities. The model takes into account the effects of preventive maintenance and accumulated load applications at the beginning of warranty a contract.

An extension of the developed reliability model is presented in Chapter 5. This extension accommodates for the effects of emergency repairs and rehabilitations on the expected number of failures. The concept of "virtual age" and the intensity reduction is relaxed to relate the effect of rehabilitation directly to the level of the design variables, and consequently to the rate of occurrence of failures.

The risk cost quantification models for three different warranty types (short-term, long-term, and maintenance warranties) are presented in Chapter 6. The model formulation includes the quantification of the expected risk cost for various warranty specifications and different rehabilitation and preventive maintenance scenarios.

The formulation and the solution approach to optimal warranty-based design and maintenance scheduling problems are presented in Chapter 7. The model formulation includes identification of the cost components in the objective functions, while the solution approach takes into account the lack of convexity for long-term design and maintenance scheduling optimization problems.

Chapter 8 presents a numerical analysis to test the applicability of the method of moments, and to illustrate the application of the developed methodology. The current AASHTO method for design of pavements is used to formulate the limit state function.

Finally, Chapter 9 summarizes major findings, addresses limitations, and presents directions for future work. This chapter also includes a discussion on positioning the problem of performance warranties in the context of the existing knowledge in the area of transportation infrastructure engineering.

CHAPTER 2 LITERATURE REVIEW

This chapter presents an overview of the background literature in four major areas pertaining to this research: warranty analysis, performance modeling, preventive maintenance and rehabilitation effects, and design and maintenance optimization. In the first section, a general background on warranty specifications is introduced. In the second section of the chapter, a brief review of probabilistic performance modeling is presented; special attention is given to reliability models and stochastic counting processes since they are frequently used in warranty analysis. In the fourth section, the approaches for modeling the effects of preventive maintenance and rehabilitation actions are reviewed, while in the fourth section, the models for design and maintenance scheduling optimization are identified.

2.1 Warranty Specifications

In spite of the abundance of technical literature for modeling warranty costs of manufactured products, to the best of the author's knowledge, there exists no documented quantitative methodology for modeling warranty costs for transportation infrastructure facilities. Previous research in transportation infrastructure warranties is mostly qualitative rather than quantitative. The Transportation Research Board (TRB, 1992) published a report summarizing construction warranties in Europe and their impacts on contracting processes. More recently, Bayraktar et al. (2004) presented the results of a comprehensive survey on current warranty practices in the U.S., and Ozbek (2004) developed a warranty clause template for the Virginia Department of Transportation's

(VDOT) road maintenance contracts. In addition these efforts, a number of seminars and conferences were organized by the Federal Highway Administration (FHWA) to discuss important issues for successful implementation of warranty contracting in the highway sector (MDOT, 2003).

In a broad legal context, a ***warranty contract*** represents a written guarantee or a contractual obligation for product's integrity and manufacturer responsibility for repair and replacement of defective parts (Garner, 2001). Warranties can be viewed from two different perspectives: those of manufacturers and of consumers. From the consumer's point of view, the purpose of warranty is to make the manufacturer liable in the event of premature failure, or an inability of the product to carry out a piece of work, fulfill a promise, or work in the proper or intended way. From the manufacturer's point of view, even though warranty contracts result in additional costs, they provide protection of the manufacturer's interests by requiring certain responsibilities on the part of the consumers.

A warranty contract is defined by: 1) the type of compensation to the customer when the product fails, 2) the dimension of the warranty, or the number of variables defining the warranty criteria, and 3) the product type, or whether the warranty takes into account a single product, group of products, or product development after the sale. Over the last three decades, a number of different warranty policies have been offered by manufacturers and studied by researchers (Blischke and Murthy, 1992).

Concerning the compensation of the consumer, there are two basic types of warranty policies: 1) the Free Replacement/Repair Warranty policy (FRW), and 2) the Pro-rata Warranty policy (PRW). Under the FRW policy, the manufacturer agrees to repair or to provide a replacement for the failed product free of charge during the period of coverage, whereas under the PRW policy, the manufacturer agrees to refund a fraction of the purchase price (Thomas and Rao, 1999). Other warranty policies can be derived from these two basic types. For example, a policy can include a free replacement up until time *T* from the initial sale, and a prorated refund for the period from *T* to the end of the coverage period. In addition, a policy can include the renewal of the coverage period every time the product is replaced; such a policy is called a renewable warranty.

Based on a number of variables defining the warranty limits, warranty polices can be classified as one- or two-dimensional (Chen and Popova, 2002). One-dimensional warranties are defined in terms of a single variable, e.g., time (*W*) or utilization (*U*), whereas two-dimensional warranties are defined as a region in a two-dimensional plane with the axes representing time and utilization. Figure 2.1 illustrates warranty intervals for one-dimensional warranty policies, while Figure 2.2 illustrates the regions for two-dimensional warranty policies.

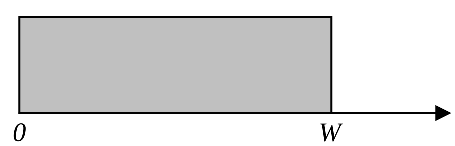
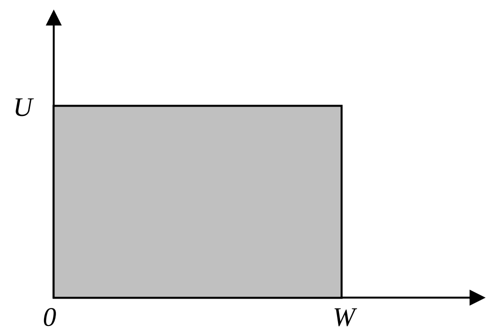
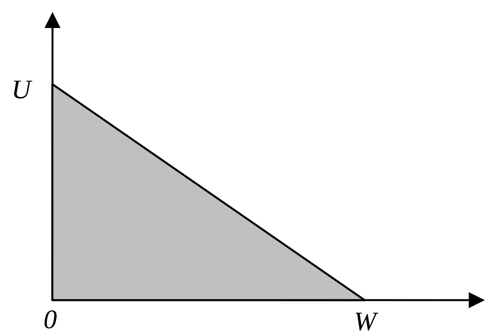


Figure 2.1 Warranty Intervals and Regions for One-dimensional Policies



b) Two-dimensional Policy (Type I)



c) Two-dimensional Policy (Type II)

Figure 2.2 Warranty Intervals and Regions for Two-dimensional Policies

Finally, warranties can be classified based on whether they apply to a single product or a group of products. Warranties applicable to a single product are referred to as Group A policies, whereas warranties applicable to a group of products are called Group B policies. There are also Group C warranty policies that include product development after the sale; these policies are mostly applied to government procurement of large and complex products, such as military equipment.

Compared to the products commonly associated with warranties, such as consumer electronics, machinery and others, transportation infrastructure is not manufactured, or built in the controlled environment of plants. Each transportation infrastructure facility represents a unique structure, constructed on subgrade with different properties and subjected to different environmental conditions. For these reasons, warranties for transportation infrastructure facilities differ from the standard manufacturing warranties.

Certainly, a unique aspect of warranty contracting for transportation infrastructure is bond requirements. Performance bonds are financial instruments used to protect the interests of the state highway agencies (SHA) in case the contractor defaults, or becomes unable to comply with the warranty terms. These bonds are obtained from surety companies that issue and price the bonds based on estimated risk. This procedure often includes evaluation of the contractor's general financial health as well as its construction quality record. Due to the lack of methods to quantify performance-related risks, a number of contractors have reported problems obtaining long-term performance bonds (MDOT, 2003).

Another unique feature of transportation infrastructure warranties is the specification of unambiguous failure criteria. Mainly due to the problem of defining a satisfactory level of performance, the concept of failure for transportation infrastructure differs from that of products commonly sold with warranties; in fact, different agencies use different performance indicators and different failure criteria. In general, there seems to be a lack of consensus about which performance indicators should be used and what should be the failure threshold.

Regardless of the type of performance indicator and its threshold, the ability to predict changes in the level of these indicators over time is of paramount importance to both the SHA and the contractors. Under the coverage of warranty, these indicators, such as distress or serviceability, are observed and recorded; if their level exceeds the threshold values, the facility is considered to fail the warranty terms, and the contractor incurs the warranty servicing costs. The only way to quantify these costs in the planning phase of the project is to relate the design and construction characteristics to the future performance; therefore, in the core of every warranty system is a model that predicts the product's performance. The following section summarizes the previous research in performance modeling.

2.2 Probabilistic Performance Modeling

It has long been recognized that the development of accurate deterioration models plays an important role in designing and managing transportation infrastructure. Due to many factors, such as simplified assumptions made for characterizing the behavior of transportation infrastructure and variability associated with material properties, the performance can never be predicted with absolute certainty; at best, it can be predicted only with the associated probability. Failing to recognize such a fact can often lead to improper design and management decisions. Models that explicitly consider uncertainties in performance prediction are often referred to as probabilistic models.

The first step in developing probabilistic models is to identify sources of uncertainty, often a daunting task for complex structural systems, such as transportation infrastructure. There are three common sources of uncertainty contributing to randomness in the utilization and structural response of a system (Oberkampf, 2001): 1) Aleatory uncertainty, or irreducible uncertainty due to an inherent irregularity in the properties and behavior of an observed system; 2) Epistemic uncertainty, or uncertainty due to a lack of knowledge about the system's behavior; and 3) Uncertainty stemming from the occurrence of both acknowledged and unacknowledged errors.

Apart from the epistemic uncertainty, two common sources of variability contribute to the uncertain response of structures: 1) material variability or the natural variation associated with the properties of materials used for construction, and 2) variability from construction process, such as the variation of as-build thicknesses of pavement layers. By considering a lump-sum effect of uncertainties on performance, researchers have extensively studied different approaches to modeling pavement and bridge deterioration. One of the most investigated approaches is based on the Markov process.

Similar to the definition of a random variable as a rule for assigning a number $z(\zeta)$ to every outcome ζ from an experiment φ, a stochastic process represents a rule for assigning a function $z(t,\zeta)$ to every outcome ζ (Papulis, 1984). A stochastic process is said to be a Markov process if the Markov assumption is satisfied. The Markov assumption states that the conditional probability of any future event is independent of the past events and depends only on the present state. The Markov process is referred to as a "memoryless" process.

Since the descriptions of many system dynamics involve differential equations that require knowledge of only the current state, not the complete history of state-transitioning, Carnahan et al. (1987) argued that the "memoryless" property is not an unreasonable assumption for pavement deterioration. Nevertheless, the validity of the

Markov assumption for transportation infrastructure deterioration has been extensively questioned (Madanat et al., 1995; Madanat et al., 1997; Mishalani and Madanat, 2002). Considering the latent nature of infrastructure deterioration, indeed, there are two possible reasons for the Markov assumption to be violated: true state dependence and “spurious” state dependence (Madanat et al., 1997). True state dependence occurs when a deterioration process is dependent on the deterioration history, whereas “spurious” state dependence or heterogeneity arises when certain unmeasured characteristics influence the deterioration process. If heterogeneity is not properly accounted for, it can lead to the conclusion that the data does not support the Markov assumption, even though the Markov property might still be present.

If the Markov assumption is not valid, the application of the Markov process is compromised; under such circumstances, generalized Markov processes and time-based models are more appropriate. Madanat and Ibrahim (1995) applied Poisson regression to estimate the generalized Markov transition probabilities. Madanat et al. (1995) and Li and Zhang (2004), respectively, used probit regression and an ordered probit model to estimate the condition state probabilities, while Madanat et al. (1997) developed a random-effect model to account for heterogeneity in data. Efforts along similar lines include the work by Mauch and Madanat (2001) to predict the distributions of the state transition times using a semi-parametric model, and the work by Prozzi and Madanat (2000) to develop a time-based Weibull model to re-estimate the parameters of the original AASHO pavement deterioration model. While these efforts indicate that substantial work has been performed to capture the latent nature of the deterioration process, these models are data-specific and do not explicitly account for the mechanics of the failure event.

Reliability models are time-based models that can distinguish only two states, failure and non-failure (survival) state. If there is a clear definition of a failure event and a consequence of the failure, such reliability models can be effectively used to predict the performance of transportation infrastructure.

Depending on how the failure event is characterized, there are two approaches to developing reliability models: 1) the approach based on a mathematical definition of the failure event, when the mechanics governing the failure are known and can be mathematically specified, and 2) the approach that uses the actual failure data from accelerated lifetime testing, when there is a lack of knowledge about the failure mechanics. The models based on the first approach are referred to as structural reliability models, whereas the models based on the second approach are called survival models. It is important to note that accelerated lifetime tests for transportation infrastructure is very expensive and limited to the local environmental conditions. In fact, these concerns often limit the application of the models developed using data from these tests.

Darter and Hudson (1973) investigated the application of structural reliability models for modeling pavement performance and provided a method for comparing the sensitivity of probability estimates with respect to each variable. Similarly, Mori and Ellingwood (1994), Troive and Sundquist (1998), Thoft-Christensen (1995), and Estes and Frangopol (2000) modeled the reliability of reinforced concrete bridge decks by specifying the mathematical models for serviceability failures. However, evaluating failure probabilities with these modeling approaches relies on either Monte Carlo simulation or the estimates of only the first two central moments of the limit state function.

In general, for warranty analysis, two types of probabilistic performance models are frequently implemented: 1) reliability, or first failure models, and 2) models that include subsequent failures and the effect of repairs and rehabilitations. While modeling the first failure includes the application of concepts from basic probability and reliability theories, modeling sequences of failures requires consideration of the theory of stochastic processes. In warranty analysis, reliability models are appropriate for short-term warranty analysis, where the effects of corrective repairs and rehabilitation are generally not considered; on the other hand, for long-term warranty analysis, where the effects of corrective repairs and rehabilitations are considered, stochastic point processes give more realistic representation of the product's performance.

2.2.1 Structural Reliability Models

One of the most traditional structural reliability methods is the stress-strength interference method (Kotz et al., 2003). This method compares a random variable that defines the level of strength and another random variable specifying the applied loads or stress; a failure occurs when the level of stress exceeds that of strength. A graphical representation of the stress-strength interference method is illustrated in Figure 2.3.

As can be observed from Figure 2.3, the failure occurs in the interference region or the overlap area of the strength and stress distributions, where the failure region is proportional to the failure probability (Sundararajan, 1995).

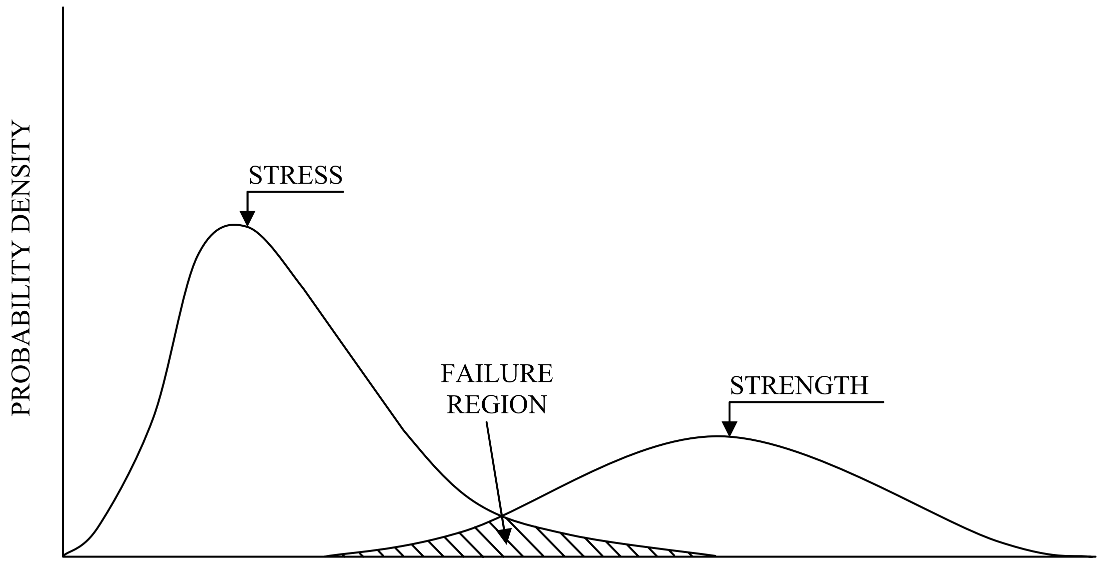


Figure 2.3 Relationship Between Stress and Strength Distributions

Even though the stress-strength interference method appears to be simple and easy to implement, its applicability depends on two factors: 1) the validity of the assumption that stress and strength are statistically independent and 2) the nature of the stress and strength distributions. If both random variables are independent and normally distributed, then there is a simple analytical solution. However, when the strength and/or stress distributions are not normal, a solution to the failure probability is not easily obtainable and requires the application of transform techniques. In addition to this computational difficulty in obtaining the failure probability, sometimes it is not appropriate to assume a specific distribution for the strength when the strength itself is a function of other random variables; rather, it is more appropriate to assume that the basic random variables in the mathematical model defining the strength are specified with known distributions. Considering the classic theory of structural reliability and the risk-based design, the stress-strength interference method represents just a simple linear case of a limit state function.

Fundamental considerations in structural reliability theory are: 1) mathematical formulation of the limit state function, 2) characterization of the basic random variables, and 3) evaluation of the multidimensional probability integral. More specifically, the structural reliability model is formulated in terms of *n* basic random variables $\mathbf{X} = [x_1, ..., x_n]^T$, and a limit state function $\mathrm{G}(\mathbf{X})$. With a defined limit state function and structural failure expressed as an event $\{\mathrm{G}(\mathbf{X}) \leq 0\}$, the probability of failure can be expressed as an *n*-dimensional probability integral:

$$\Pr[\mathrm{G}(\mathbf{X}) \leq 0] = \int_{\mathrm{G}(\mathbf{X}) \leq 0} f(\mathbf{X}) d\mathbf{X} \tag{2.1}$$

where $f(\mathbf{X})$ represents the joint probability density function of the basic random variables in vector $\mathbf{X}$. .

Since evaluation of the integral defined in Equation 2.1 can be a challenge, various approximation techniques such as Monte Carlo simulation (MCS) and the first order reliability method (FORM) are commonly used (Madsen et al., 1986). Even though these methods are often presented as competing methods of integration, Bjerager (1991) studied their compatibility. In particular, Bjerager identified problems where MCS is preferred over the FORM and vice versa; even though MCS is applicable to a wider range of problems, it is computationally intensive and cannot be easily implemented in mathematical programming. To conduct MCS, the minimum sample size to get a probability estimate with a confidence level is $100/p$, where p represents failure probability.

In contrast to MCS, the FORM is an analytical method based on linear approximation of a nonlinear limit state at the design point. In the FORM, the design point is a point on the limit state curve that is closest to the origin; the actual distance between those two points indicates a measure of reliability, often referred to as a reliability index β. Figure 2.4 illustrates a simple two-dimensional linear limit state function $(G(X) = X_1 - X_2)$ with two normal basic random variables (X_1, X_2), where the Hasofer-Lind reliability index β_{HL} is defined as:

$$\beta_{HL} = \frac{\mu_{G(\boldsymbol{X})}}{\sigma_{G(\boldsymbol{X})}} = \frac{\mu_{X_1} - \mu_{X_1}}{\sqrt{\sigma_{X_1}^2 + \sigma_{X_2}^2}} \tag{2.2}$$

With the estimated value of the reliability index, the failure probability can be expressed as:

$$prob[G(X) \leq 0] = \Phi(-\beta_{HL}) \tag{2.3}$$

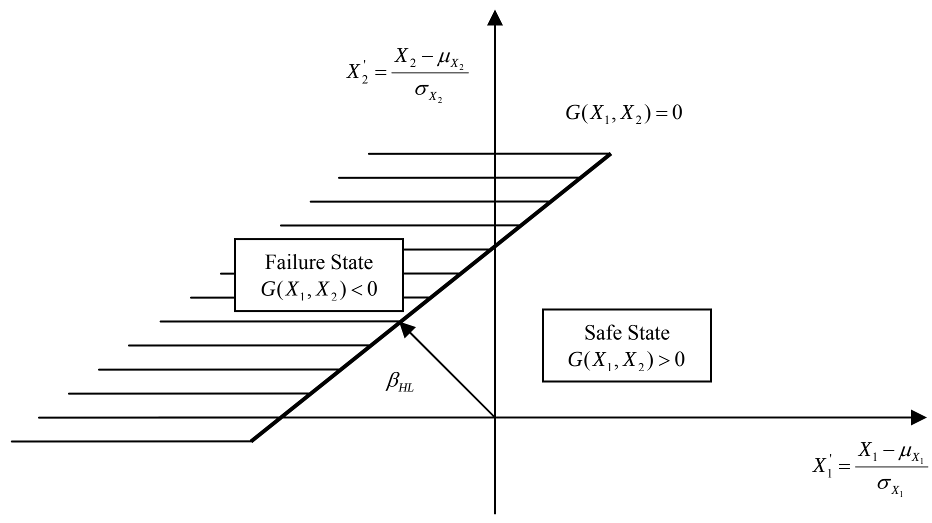


Figure 2.4 Hasofer-Lind Reliability Index for Linear Limit State Function

The main advantage of the FORM is its analytical traceability and satisfactory level of accuracy even for extremely small probabilities. On the other hand, the FORM has its shortcomings too, such as the lack of accuracy for highly nonlinear limit state functions and the difficulty in its iteration-based process of searching for the design point. Although some of the FORM shortcomings can be addressed by using the second-order reliability method (SORM), the first-order third-moment method (FOTM) (Tichy, 1994), and genetic algorithms, its inherit problem with nonlinear iteration algorithm in searching for the design point is still difficult to overcome (Zhao and Ono, 2001).

Recently, an improved approach, based on the relationship between the higher-order central moments and the failure probability, was investigated by Zhao and Ono (2000; 2004). The main advantages of this method of moments (MM) are its computational simplicity, analytical traceability, and satisfactory level of accuracy even for highly non-linear limit state functions. While Zhao and Ono (2001) report great accuracy of the method of moments, the developed models still consider only static or time-independent formulation of the limit state function.

Since the reliability of transportation infrastructure represents a probability that a facility will perform its intended function over a period of time, the measures of reliability should be dependent on time. For example, where the wear-out deterioration process is the primary cause of failures, the stress is often considered to be time-dependent; as time passes, the level of accumulated damage or the level of load applications increases. In addition to the stress, the strength can also be dependent on time. Bilikam (1985) suggested that distribution parameters of any basic random variable in the limit state function could be defined as time-dependent. Similarly, Basu and Ebrahimi (1983) assumed that both the stress and the strength are either a Weibull or a Wiener processes, whereas Madsen et al. (1986) presented a limit state formulation where the stress is a normal process.

2.2.2 Stochastic Counting Processes

Stochastic point processes represent a natural extension of reliability models in which the effects of emergency repairs and rehabilitations can be considered (Høylan and Rausland, 1994). Each time the failure occurs, emergency repair is performed, and the system is restored to a functioning state. The time between two consecutive failures is referred to as the inter-arrival time. A stochastic point process describes a sequence of these inter-arrival times.

A random variable of special interest in warranty analysis is the number of failures $N(t)$ during a warranty period W. Since it specifies the number of failures, the process $[N(t), t \geq 0]$ is referred to as a count process; more formally, a stochastic process $[N(t), t \geq 0]$ is said to be a counting process if $N(t)$ satisfies the following four assumptions (Ross, 1983):

a) $N(t) \geq 0$,

b) $N(t)$ is integer,

c) If $s < t$ then $N(s) \leq N(t)$,

d) For $s < t$, $[N(t) - N(s)]$ represents the number of failures in interval $(s, t]$.

Three types of counting processes are commonly used in warranty analysis: 1) the homogenous Poisson processes (HPP), 2) the renewal processes, and 3) the non-homogeneous Poisson processes (NHPP). The HPP represents a process in which all the inter-arrival times are independent and exponentially distributed. Similarly, the renewal process is a point process where the inter-arrival times are also independent, but not exclusively exponentially distributed; rather the inter-arrival times for the renewal process can be specified with an arbitrary probability distribution. In this context, the HPP is just a special case of the renewal process. In contrast to the HPP and the renewal process, the NHPP is a process where the inter-arrival times are neither independent nor identically distributed. In the NHPP, the emergency repairs leave the system in an as-bad-as-old state. This assumption is a realistic assumption for complex systems where the emergency repair affects only one part of the system, but does not change the overall trend of the rate of occurrence of failure (ROCOF) function. Since transportation infrastructure facilities can be described as systems composed of many interacting components, the application of the NHPP in warranty analyses is an appropriate representation of the performance. For example, a localized pavement failure might prompt a local patching, an action that does not improve the structural condition, nor changes the overall deterioration process.

2.3 Effects of Preventive Maintenance and Rehabilitations

Preventive maintenance and periodic rehabilitations play an important role in warranty analysis for transportation infrastructure. With the planned application of preventive maintenance and periodic rehabilitation, the deterioration process can be abated, and the service life expanded. In contrast to emergency repairs, which are applied when the facility fails, these actions are proactive in nature and applied before facility fails.

The effects of preventive maintenance and rehabilitations on facility performance are different. For example, preventive maintenance actions, such as pavement seal coats, can decrease the pavement roughness; however, do not significantly reduce the future deterioration intensity. On the other hand, rehabilitative actions (rehabilitations), such as pavement overlays, in addition to decreasing the current roughness condition, can also reduce the future deterioration intensity.

The effects of preventive maintenance and rehabilitations are commonly modeled using transition probabilities (Carnahan et al., 1987) for a stochastic Markov Decision Process (MDP), using roughness improvement function for a deterministic MDP (Tsunokawa and Schofer, 1994), and using the condition factor and the remaining life concept (AASHTO, 1986) for modeling deterioration of the structural number (*SN*).

Following the publication of the American Association of State Highway and Transportation Officials (AASHTO) pavement design guide in 1986, the validity of the remaining life concept, introduced in this guide, was extensively questioned by Elliot (1989) and Fwa (1991). As a result, in its 1993 pavement design guide, the AASHTO has changed the rehabilitation guidelines by setting the value of the remaining life coefficient to be one (AASHTO, 1993). More recently, Abaza (2005) summarized the state-of-the-art models for design of pavement overlays.

As previously discussed, a random variable of special interest in warranty analysis is the number of failures during the coverage of warranty. Stochastic counting processes provide a statistical description of this random variable. The fundamental assumption of the emergency repair efficiency for counting processes is either the minimal repair efficiency, when a repair action leaves the system's state in as-bad-as-old condition, or the perfect repair efficiency, when a repair action restores the system to as-good-as-new condition. These emergency repairs, respectively, correspond to the NHPP and the renewal process. However, in reality, the effects of preventive maintenance and rehabilitations on transportation infrastructure are neither minimal nor perfect.

Since the efficiency of rehabilitations is neither minimal nor perfect, the effects of rehabilitations can be modeled as imperfect (Lin et al, 2000; Doyen and Gaudion, 2002). The literature review identifies two general approaches to modeling imperfect repairs, one based on assigning the probabilities to the two extreme cases of repair efficiency, and the other based on controlling the failure intensity function, or the rate of occurrence of failure (ROCOF) function. In the Brown-Proschan model, the system's state after repairs is assumed to be as-good-as-new with a probability p and as-bad-as-old with a probability $1-p$. By considering the direct effect of rehabilitations on the ROCOF function, Doyan and Gaudion (2002) developed two arithmetic reduction models: the arithmetic reduction of intensity (ARI) model and the arithmetic reduction of age (ARA) model.

The ARI model describes a situation when rehabilitation causes a one-time reduction in the intensity of the ROCOF function; following this, the ROCOF function continues to increase with the same rate as before the rehabilitation is taken. On the other hand, the ARA model represents a situation in which rehabilitation resets the ROCOF function to zero and causes a reduction in the effective, or the virtual age of the product; following this, the ROCOF function is defined by the effective virtual age of the product. Figure 2.5 illustrates the difference between the ARI and the ARA modeling approaches.

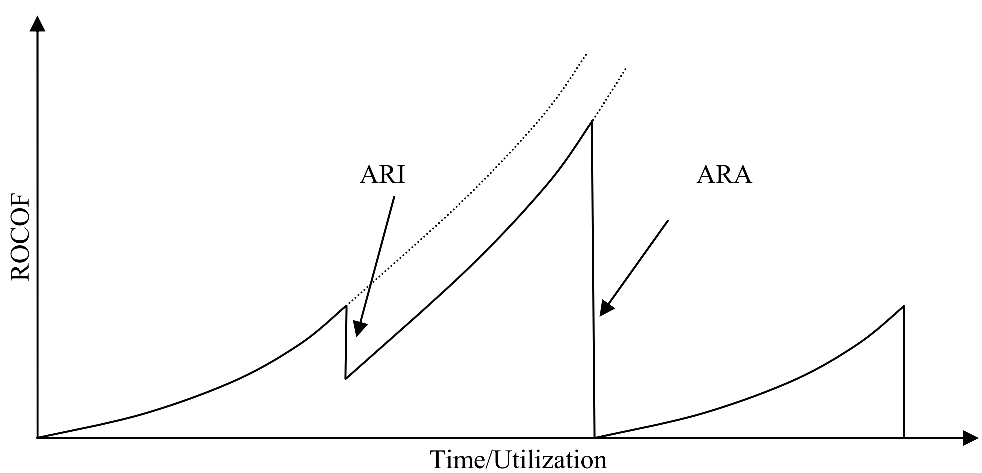


Figure 2.5 Effects of ARI and ARA Types of Rehabilitation on the ROCOF

An inherent problem with both the Brown-Proschan and the Doyan-Gaudoin models is their lack of ability to consider an important class of rehabilitation effects:

when rehabilitation leaves the system in a better-than-new state. Many important types of transportation infrastructure facilities receive such rehabilitations. For example, the application of thick overlays leads to an increase in pavement structural capacity to a level above the initial one; similarly, bridge retrofitting can lift the bridge deck condition to a better-than-new state.

2.4 Design and Maintenance Optimization

In the field of transportation infrastructure engineering, optimization models are typically classified in two categories: the models in which the goal is to determine the optimal design, and the models in which the goal is to determine the optimal schedule of preventive maintenance and rehabilitations. In general, for both network and project level analysis, decision-makers are faced with several problems, such as finding the optimal design strategy, maximizing the reliability, determining the optimal rehabilitation frequency, and estimating the minimum cost for operating the facility/network over the planning period. Mathematical programming models provide an insight into cost effectiveness of different design and rehabilitation strategies.

At least from the theoretical point of view, the reliability-based structural design represents a well-established concept (Thoft-Christensen, 2000). Since the reliability of a transportation infrastructure facility can be immensely different for different material types, structural topology and configuration, the ultimate goal of reliability-based structural design is to find the best possible design solution.

There are many ways to formulate the optimal reliability-based structural design problem; some of the important problem formulations are as follows: minimize the cost of design subjected to the reliability and structural constraints, maximize the reliability of design subjected to the cost and structural constraints, and minimize the initial cost plus the expected cost of failures subjected to the structural constraints. In this context, two design philosophies can be distinguished: 1) design on the basis of a fixed probability of failure, or in other words, design by a given probability of exceedance, and 2) design based on economic optimization of the life-cycle costs.

Even though both design philosophies represent a valid approach to transportation infrastructure design, from the perspective of warranty analysis and product development, design based on economic optimization of the life cycle costs is more suitable. This is due to the fact that the design problem based on a fixed probability of failure is highly sensitive to the probability of exceedance constraint. In general, there seems to be an agreement in the research community that the optimal design should not be based only on minimizing design and construction costs, but should also include maintenance and the expected failure costs (Thoft-Christensen, 2000).

Wen (2001) formulated the life-cycle design problem for designing structures in different geographical areas with different seismic hazards, while Klatter and Noortwijk (2003) considered a life-cycle approach to bridge management. Madanat et al. (2002) accounted for the effects of performance model accuracy on the optimal pavement design, and Blischke and Murthy (2000) discussed general optimization models for products sold with warranties.

The literature review also revealed the extensive work in the field of determining optimal rehabilitation strategies for both transportation infrastructure facilities and transportation networks. Carnahan (1988) formulated the problem of finding the optimal pavement rehabilitation action as the Markov Decision Problem (MDP), while Madanat and Ben-Akiva (1994) included measurement errors and formulated the problem as a latent Markov decision model. Recently, Ouyand and Madanat (2004) developed a mixed-integer programming formulation for determining the optimal rehabilitation decisions for pavement during a planning period.

Similar efforts have been reported for other transportation infrastructure facilities. Redmond et al. (1997) developed a model for determining the optimal bridge renovation time, while Stewart et al. (2004) formulated a problem of bridge deck replacement for minimum expected cost under multiple reliability constraints. Since warranty analysis represents a new development in construction and management of transportation infrastructure facilities, to the author's best knowledge, there are no published studies for determining the optimal schedule of rehabilitations during the warranty period; nevertheless, the literature review reveals a substantial research effort for studying the optimal preventive maintenance schedule for general products sold with warranties.

Chun (1992) considered a problem of determining the optimal number of rehabilitations by minimizing the expected cost of repairs and rehabilitations over the warranty period. In Chun's model, the effect of rehabilitation is modeled based on the fixed-age reduction assumption. Keeping the same assumption, Jack and Dagpunar (1994) showed that a strict periodic rehabilitation strategy is not the optimal strategy, if the product has an increasing failure rate. They showed that for a warranty period W, and with the rehabilitation effectiveness specified with age reduction x, the optimal strategy is to perform n rehabilitation actions at intervals x apart, followed by the final interval $W - nx$. Dagpunar and Jack (1994) and Jack and Murthy (2002) relaxed the fixed-age reduction assumption and developed a model where the amount of age reduction is under the control of the manufacturers.

2.5 Summary

This chapter presents the literature review relevant to the overall objectives and introduces the necessary background to analyze performance warranty contracts for transportation infrastructure. The literature review has revealed a lack of comprehensive methodology for the quantitative analysis of transportation infrastructure warranties, yet has shown substantial developments in related fields. In the following chapter, the models from the literature on warranty specifications for manufactured products are modified and adapted for characterizing warranty systems for transportation infrastructure.

CHAPTER 3 INTEGRATED FRAMEWORK FOR STUDY OF PERFORMANCE WARRANTIES

To study performance warranties for transportation infrastructure, a framework aiming to integrate their various aspects is required. This framework needs to be generic and easily adjustable to account for the different types of transportation infrastructure as well as to reflect local experience and knowledge. This chapter introduces such a framework. First, a system representation of performance warranties is developed and its components are characterized. Following the discussion of the system's characterization, a research methodology is presented in the next section. This research methodology provides a blueprint for model development, system analysis, and result interpretation.

3.1 Warranty System

Much like theories for studying social or natural phenomena, systems theory also provides an integrated framework for studying warranties. A systems approach is a general approach for studying real-world problems using an abstract representation of the observed phenomenon, independent of its forms. Applied to the study of warranties, this approach involves four steps (Murthy and Blischke, 1992):

a) System characterization;

b) Model development;

c) Analysis of the system; and

d) Interpretation and application of the results.

The warranty system characterization is a process in which the system is simplified by defining the inter-dependent factors influencing its behavior. Since each factor interacts with many other factors and also involves many different variables, depending on the nature of the variables, the same system can be characterized in many different manners. For example, the system could be characterized as deterministic or stochastic, dynamic or static, etc.

Once the system is characterized, a descriptive model can be developed and used to study warranties. However, without mathematical representation of the system, this study would be limited to a mere qualitative analysis. As noted in Chapter 2, currently most of the warranty studies for transportation infrastructure are qualitative, rather than quantitative. To shift from a qualitative to a more quantitative analysis, mathematical models of the warranty system need to be developed. The mathematical modeling is a process in which the system characteristics or descriptive models are transformed into mathematical models by linking the variables and inter-dependent factors in an abstract mathematical formulation.

The next step in the study of warranties is the analysis of the characterized system. This is a process where standard mathematical tools and techniques are used to evaluate the system's behavior. An example of such an analysis is quantification of the risk cost, or the warranty servicing cost for different specifications of the model parameters. Finally, the last step of the systems approach to studying warranties includes interpretation and application of the analysis results. This step involves restoring the one-to-one correspondence of the numerical results to the inter-dependent factors defined in the descriptive model. With such a restored connection, decision-makers can decide on the actions that will yield the desired effect. However, it is important to note that for the effective implementation of the systems approach, it is essential that the developed descriptive models accurately represent all the relevant processes and factors in the warranty system. In the following section, the most important inter-dependent factors are closely examined.

3.1.1 System Characterization for Performance Warranties

As previously discussed, a systems approach to studying warranties can be effectively used only if a descriptive model of the system is appropriately specified. To develop an appropriate descriptive model, the first step is to identify the inter-dependent factors influencing system behavior. Once these factors are identified, the next step in system characterization is to define the relationships among them and to specify the system's variables. Murthy and Blischke (1992) analyzed the factors influencing general warranty systems and developed a framework for determining the relationship among them.

The most important factor in any warranty analysis is the specification of the warranty policy. As noted in Chapter 2, the warranty policy for transportation infrastructure can be specified with: 1) the dimension of the warranty, 2) the type of failure mode and the failure criterion/criteria, 3) the consequence of the failure event, and 4) the responsibility for the application of preventive maintenance and rehabilitations. As such, performance warranties for the procurement and management of transportation infrastructure can be classified into three categories (FHWA, 2003): short-term warranties, long-term warranties, and maintenance warranties.

Short-term performance warranties are warranties that are implemented as a safeguard against the risk of latent flaws and defects, hidden in the design and construction phases. Typically they range from two to ten years after construction is completed and consider only the application of preventive maintenance actions during the coverage of the warranty. Figure 3.1 illustrates the life-cycle phases included in short-term performance warranties.

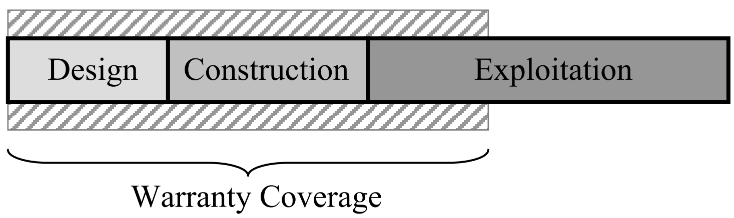


Figure 3.1 Short-term Performance Warranty Specifications

Even though short-term performance warranties provide some degree of protection against poor performance, they cover only a short period of the facility's life-cycle. To shift all the performance-related risks to the contractors, state highway agencies (SHA) can implement ***long-term performance warranties***. These warranties cover the entire life-cycle and allow for the application of both preventive maintenance and rehabilitations. The life-cycle phases involved with long-term performance warranties are illustrated in Figure 3.2.

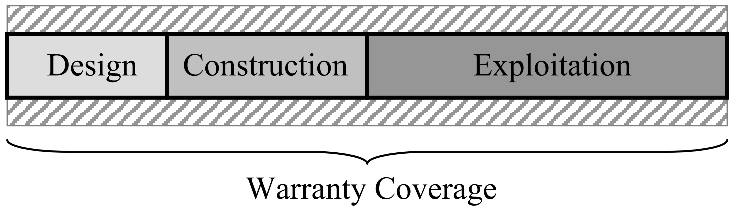


Figure 3.2 Long-term Performance Warranty Specifications

In contrast to the previously discussed performance warranties that involve design and construction phases, ***maintenance performance warranties*** cover only the exploitation phase of the transportation infrastructure life-cycle. Maintenance performance warranties consider the application of both preventive maintenance and rehabilitations and are also sometimes referred to as performance-specified maintenance contracts. Figure 3.3 shows involvement of the life-cycle phases for maintenance performance warranties.

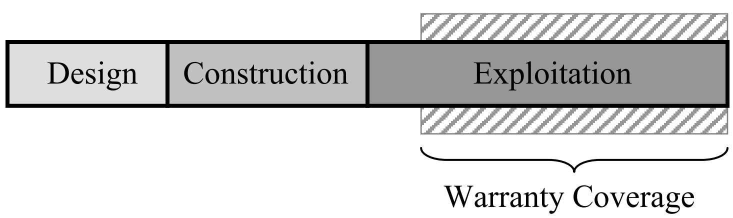


Figure 3.3 Maintenance Performance Warranty Specifications

In addition to the specification of the warranty policy, the warranty system includes many other important inter-dependent factors. Figure 3.4 illustrates a simplified warranty system for the three common categories of warranty specifications. As can be observed from Figure 3.4, there are general and specific characteristics of the warranty system. The general factors influencing the warranty system for transportation infrastructure are as follows: agency, contractor, design, construction, and performance.

The ***agency*** or the owner factor indicates the interest of a customer/owner to protect its exposure to unsatisfactory performance of a procured or managed facility by specifying the terms of the warranty policy. Therefore, from the perspective of the agency, the objective is to procure and manage transportation infrastructure with the minimal total costs. Depending on the type of contracting method, the agency can either specify the design, or let the contractor choose it. In the general warranty system characterization section in Figure 3.4, a scenario where the agency specifies the design is indicated with a dashed line.

In contrast to the agency, the ***contractor*** represents a factor that takes into account the perspective of a company performing the design, construction, and/or maintenance work. From the perspective of the contractors, the objective is to minimize the construction cost and the warranty servicing costs as well as to use the warranties as a marketing tool. Again, depending on the type of contracting method, the contractor could be either required to build a facility based on a design specified by the agency, or allowed to design and subsequently build the facility.

The ***design*** factor corresponds to the process in which the facility is designed, whereas the ***construction/maintenance*** factor refers to the processes in which the facility is constructed and/or maintained. Depending on the type of contracting method, these two factors can be directly liked to the contractor, as in the case of the design-build method. Finally, after the facility is designed, constructed, and put in service, its

performance is observed. If the facility does not meet the warranty specifications, the contractor incurs the warranty servicing cost. If the facility is well-designed and constructed, it should be resistant to failures, and the warranty servicing cost will be relatively small.

In addition to the general characteristics of the system, there are specific characteristics of the warranty system. In short-term warranty analysis, rehabilitations are not considered; therefore, reliability models are appropriate models for performance prediction. There are two cost components contributing to the overall costs of short-term warranties: the risk cost or the warranty servicing cost, and the construction costs. The optimal design can be determined by minimizing the sum of these two cost components. Because of the nonlinearity in reliability models, the optimal warranty-based design problem is inherently a nonlinear optimization problem.

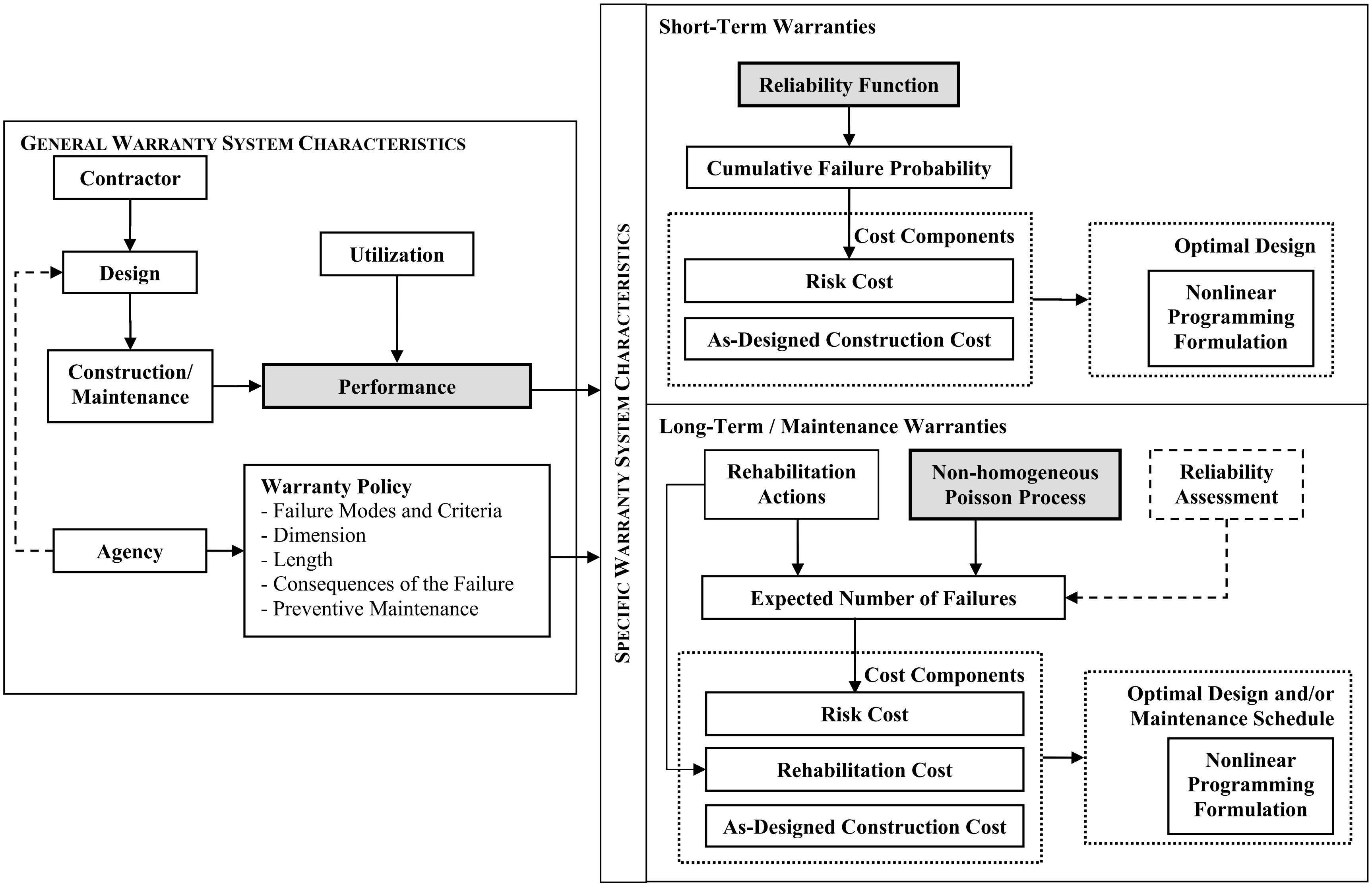


Figure 3.4 Performance Warranty System Characterization

In long-term warranties, rehabilitations are considered in the analysis, and therefore, as discussed in Chapter 2, the non-homogeneous Poisson process can be used to develop the corresponding performance prediction models. There are three cost components contributing to the total costs of long-term warranties: the risk cost or the warranty servicing cost, the construction cost, and the cost of rehabilitations. The optimal design strategy for long-term warranties can be determined by minimizing the total cost.

Finally, maintenance warranties can be viewed as a special type of long-term warranty, where the construction has been completed by other contractors; hence, the construction cost does not contribute to the total cost. Since the facility is already in service, the performance models need to take into account the effect of the existing reliability by determining the accumulated load applications at the beginning of a warranty contract. In Figure 3.4, the influence of this factor is considered as part of the specific characteristics of long-term/maintenance warranties and indicated with a dashed line.

3.2 Research Methodology

With the developed descriptive model of the warranty system, the next step is to develop a mathematical representation of the system. Figure 3.5 illustrates the research process used for the mathematical modeling and analysis of the warranty system. First, the design inputs in terms of design criteria and local characteristics are specified. For example, the failure criteria and in-place material properties might be considered as such inputs. Second, the mean values of the design parameter are determined by solving the design equation for the mean values of the design inputs and the predicted load applications for the entire design period. Third, the "stress" is specified directly from the process of predicting future load applications with a functional model of basic random variables and time. Fourth, with the "strength" and the "stress" defined, the limit state function is defined as the difference between the strength and the time-dependent stress. Fifth, using the characterized variability of the basic random variables, the time-dependent multidimensional probability integral is evaluated using both Monte Carlo simulation (MCS) and the method of moments. Then, the estimates of the failure probability using MCS and the method of moments are compared and a decision is made regarding the applicability of the method of moments to modeling the reliability. Finally, the models for quantifying the risk cost and determining the optimal warranty–based design and maintenance schedule are developed.

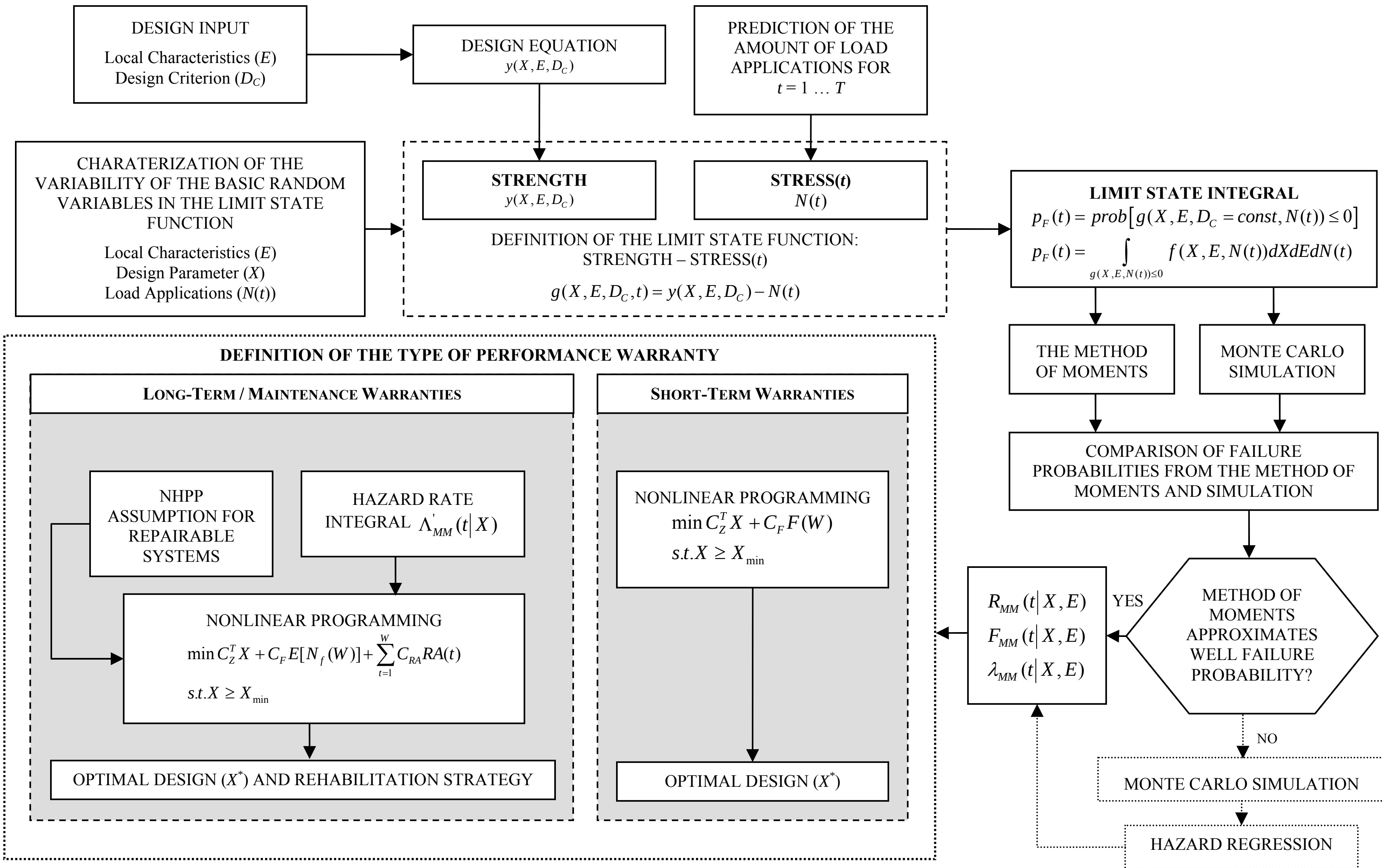


Figure 3.5 Research Methodology

3.3 Summary

This chapter presents an integrated framework for the study of warranties. The relationships among the significant factors influencing the warranty system are identified and discussed. Based on the identified inter-dependent factors, the descriptive models for three different types of transportation infrastructure performance warranties are developed. Finally, a research methodology for the mathematical modeling and analysis of warranties is presented. In the following chapter, the reliability models for warranty analysis are developed.

CHAPTER 4 RELIABILITY MODELS FOR WARRANTY ANALYSIS

As discussed in Chapter 2, when the mechanics of a failure event are known and can be mathematically specified, structural reliability models can be effectively used to estimate the reliability of transportation infrastructure facilities. This chapter presents a mathematical specification for developing reliability functions based on the method of moments. In the first section, time-dependent limit state functions are discussed. In the second section, a point estimation technique for determining the high-order central moments of the limit state function is introduced; then, in the third section, three standardizing functions for developing reliability functions are developed. Finally, in the last two sections, two extensions of the reliability model are discussed: one extension that considers the reliability assessment is presented in the fourth section, while the other extension that considers the application of preventive (routine) maintenance is presented in the fifth section.

4.1 Time-Dependent Limit State Function Formulation

For deterioration processes involving fatigue and wear-out mechanics, the strength is commonly specified with a function defining the allowable number of load applications. To compare stress and strength, the corresponding function indicating the time-dependent stress needs to be specified in terms of the accumulated load applications, or the consumed strength.

The concept of time-dependent limit state functions is illustrated in Figure 4.1. It can be observed from the figure that over time the overlap between the time-dependent stress and strength probability densities increases. This overlap indicates the failure region and is directly proportional to the failure probability.

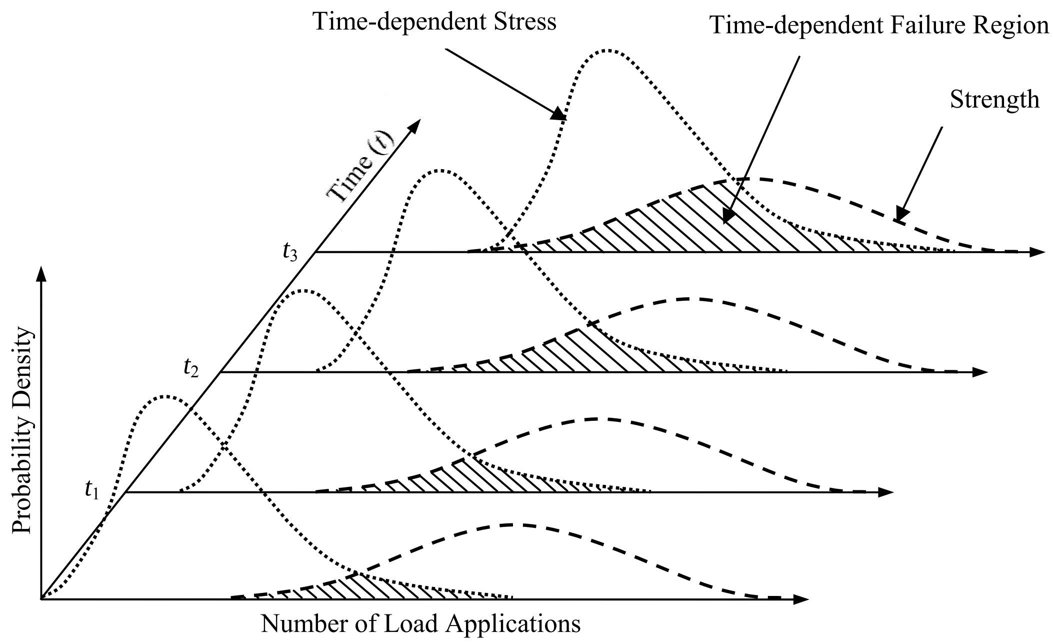


Figure 4.1 Time-dependent Strength-Stress Interference

Mathematically, a time-dependent limit state function $G(\mathbf{x},\mathbf{y},t)$ can be defined as follows:

$$G(\mathbf{x},\mathbf{y},t) = q(\mathbf{x}) - z(\mathbf{y},t) \tag{4.1}$$

where,

$q(\cdot)$ - function defining the strength, or the allowed number of load applications,

$z(\cdot)$ - function defining the stress, or the accumulated load applications,

$\mathbf{x} \in \mathbb{R}^n$ - vector of n basic random variables in the strength function, and

$\mathbf{y} \in \mathbb{R}^m$ - vector of m basic random variables in the stress function.

Without loss of generality, the basic random variables are assumed to be mutually independent. If there is an indication that the basic random variables are correlated, the orthogonal transformation can be applied to transform the correlated variables into their corresponding uncorrelated counterparts.

With the defined limit state function and the structural failure expressed as an event $\{G(\mathbf{x},\mathbf{y},t) \leq 0\}$, the probability of failure at time t can be expressed as an (n+m)-dimensional probability integral:

$$F(t) = \Pr[G(\mathbf{x},\mathbf{y},t) \leq 0] = \int_{G(\mathbf{x},\mathbf{y},t)\leq 0} f(\mathbf{x},\mathbf{y},t)d\mathbf{x}d\mathbf{y} \tag{4.1}$$

where $f(\mathbf{x},\mathbf{y},t)$ represents the joint probability density function of the basic random variables at time t.

As noted in the literature review, an accurate and analytically tractable solution to the failure probability integral specified in Equation 4.1 cannot be easily obtained. To improve accuracy, but still maintain the closed-form solution, the reliability indices need to be estimated using the information from the high-order moments of the limit state function. A method based on this approach is referred to as the method of moments (Zhao and Ono, 2000).

The method of moments is based on two sequential steps. First, to allow for more estimating points and improve the accuracy of the calculated central moments, the moments of the limit state function are determined using the point estimates obtained in the standard normal space. Second, after the moments of the limit state function are obtained, the reliability indices and the failure probabilities are estimated using the existing standardization functions.

4.2 Central Moments of Time-Dependent Limit State Function

Mathematically, statistical moments of the function of random variables $G(\mathbf{x}, \mathbf{y}, t)$ can be expressed as:

$$\mu_G(\mathbf{x}, \mathbf{y}, t) = \int G(\mathbf{x}, \mathbf{y}, t) f(\mathbf{x}, \mathbf{y}, t) d\mathbf{x} d\mathbf{y} \tag{4.2}$$

$$M_{kG}(\mathbf{x}, \mathbf{y}, t) = \int \left(G(\mathbf{x}, \mathbf{y}, t) - \mu_G \right)^k f(\mathbf{x}, \mathbf{y}, t) d\mathbf{x} d\mathbf{y} \qquad \text{for } k \geq 2 \tag{4.3}$$

where μ_G represents the mean and M_{kG} the *k*-th central moment of $G(\mathbf{x}, \mathbf{y}, t)$.

Traditionally, the Taylor expansion method was used to evaluate Equations 4.2 and 4.3; however, this approach involves computation of high-order derivatives which can often be a daunting task for complicated limit state functions. To avoid computation of derivatives, the central moments can be calculated as a weighted sum of the limit state function at a finite number of points chosen to satisfy the following condition (Rosenblueth, 1975; Zhao and Ono, 2000):

$$M_{kx} = \sum_{j=1}^{J} P_j (x_j - \mu_x)^k \tag{4.4}$$

where x_j represents the *j*-th estimating point and P_j the corresponding weight.

For more accurate estimation of the moments, it is necessary to have many estimating points. Since the estimating points and their corresponding weights for standard normal variables can be directly obtained from the Hermite integration, it is convenient to transform the original basic random variables to the standard random variables.

For a function with only one random variable, using the inverse Rosenblatt transformation, the condition specified in Equation 4.4 can be rewritten as:

$$M_{kx} = \sum_{j=1}^{J} P_j \left(G[T^{-1}(u_j)] - \mu_G \right)^k \tag{4.5}$$

where $T^{-1}(u_j) = F^{-1}\left[\Phi(u_j)\right]$ represents the inverse Rosenblatt transformation at the estimating point u_j with the corresponding weight P_j, F the cumulative distribution function of the random variable under consideration, and Φ the cumulative standard normal probability.

For example, for a five-point estimate of the moments (Zhao and Ono, 2000), the estimating points $u_1,...,u_5$ and corresponding weights $P_1,...,P_5$ are readily obtainable and summarized in Table 4.1 (Abramowitz and Stegum, 1972).

Table 4.1 Five-Point Estimates in Standard Normal Space

Point Number (j)	Estimating Point u_j	Corresponding Weight P_j
1	0	0.53333
2	1.35563	0.22208
3	-1.35563	0.22208
4	2.85697	0.01126
5	-2.85697	0.01126

This procedure can be further generalized to consider functions of many random variables. Since the limit state function with (n+m) basic random variables considered at l estimating points requires $(n+m)^l$ evaluations, the problem of calculating central moments becomes computationally intensive. For example, if there are 10 basic random variables defining the limit state function, with 5 estimating points, a total of 10^5 evaluations will be needed to estimate the moments. To avoid this computation problem, the limit state function $G(\mathbf{x},\mathbf{y},t)$ can be approximated as follows (Zhao and Ono, 2001):

$$G^*(\mathbf{x},\mathbf{y},t) = G^* = \sum_{i=1}^{(n+m)} (G_i - G_\mu) + G_\mu \tag{4.6}$$

where $G_\mu = G(\mu)$ represents the original limit state function evaluated at the mean level of all basic random variables and $G_i = G\left[T^{-1}(u)\right]$ is the original function in which all the basic random variables take their mean values, except for the *i*-th variable, which is represented with the inverse Rosenblatt transformation of the estimating point *u*.

Since G_i represents a function of only one basic random variable, its central moments can be calculated as follows:

$$\mu_{G_i} = \sum_{j=1}^{J} P_j G_i[T^{-1}(u_j)] \tag{4.7}$$

$$\sigma_{G_i}^2 = \sum_{j=1}^{J} P_j \left\{ G_i[T^{-1}(u_j)] - \mu_{G_i} \right\}^2 \tag{4.8}$$

$$\alpha_{rG_i}\sigma_{G_i}^r = \sum_{j=1}^{J} P_j \left\{ G_i[T^{-1}(u_j)] - \mu_{G_i} \right\}^r \tag{4.9}$$

where μ_{G_i}, $\sigma_{G_i}^2$, α_{rG_i} represent the mean, variance, and *r*-th dimensionless central moments of G_i, respectively, T^{-1} the inverse Rosenblatt transformation, and $u_1,...,u_J$ the estimating points with $P_1,...,P_J$ as the corresponding weights.

Finally, the first four time-dependent central moments of the approximated limit state function $G^*(\mathbf{x},\mathbf{y},t) = G^*$ can be obtained as follows:

$$\mu_{G^*}(\mathbf{x},\mathbf{y},t) = \sum_{i=1}^{(n+m)} (\mu_{G_i} - G_\mu) + G_\mu \tag{4.10}$$

$$\sigma_{G^*}^2(\mathbf{x},\mathbf{y},t) = \sum_{i=1}^{(n+m)} \sigma_{G_i}^2 \tag{4.11}$$

$$\alpha_{3G^*}(\mathbf{x},\mathbf{y},t) = \frac{\sum_{i=1}^{(n+m)} \alpha_{3G_i}\sigma_{G_i}^3}{\sigma_{G^*}^3} \tag{4.12}$$

$$\alpha_{4G^*}(\mathbf{x},\mathbf{y},t) = \frac{\sum_{i=1}^{(n+m)} \alpha_{4G_i}\sigma_{G_i}^4 + 6\sum_{i=1}^{(n+m)-1}\sum_{b>i}^{(n+m)} \sigma_{G_i}^2\sigma_{G_b}^2}{\sigma_{G^*}^4} \tag{4.13}$$

Equations 4.10 – 4.13 require only $(m+n)l$ evaluations of the limit state function, which represents a significant reduction in the number of evaluations. This is a very important reduction if more estimation points are considered; a marginal increase in the number of estimating points would increase the number of evaluations in a multiplicative rather than in an exponential manner.

4.3 Standardization Functions for Estimating Reliability

Standardization functions are functions used for transforming random variables to standard normal variables. The most common standardization function is a well-known function used to transform normal random variables to standard normal variables. If the limit state function $G(\mathbf{x},\mathbf{y},t)$ is normally distributed, the estimates of mean and variance would be sufficient to calculate the reliability. Under such a condition, the reliability index $\beta(t)_{2M}$ and the failure probability $F(t)_{2M}$ can be determined using the following equations:

$$\beta(\mathbf{x},\mathbf{y},t)_{2M} = \frac{\mu_{G^*}(\mathbf{x},\mathbf{y},t)}{\sigma_{G^*}(\mathbf{x},\mathbf{y},t)} \tag{4.14}$$

$$F(\mathbf{x},\mathbf{y},t)_{2M} = \Phi(-\beta(\mathbf{x},\mathbf{y},t)_{2M}) \tag{4.15}$$

However, when $G(\mathbf{x},\mathbf{y},t)$ is not normally distributed, Equations 4.14 and 4.15 can significantly either underestimate or overestimate the reliability (Kotz et al., 2003). To overcome this problem, the third moment (skewness) and the fourth moment (kurtosis) need to be considered.

With available estimates of the third moment of the limit state function, assuming that the standardized variable follows a three-parameter lognormal distribution, the third-moment reliability index (3M) and the failure probability can be expressed as (Zhao and Ono, 2001):

$$\beta(\mathbf{x},\mathbf{y},t)_{3M} = \frac{-sgn(\alpha_{3G^*}(\mathbf{x},\mathbf{y},t))}{\sqrt{\ln(A)}} \ln\left[\sqrt{A}\left(1+\frac{\beta(\mathbf{x},\mathbf{y},t)_{2M}}{u_b}\right)\right] \tag{4.16}$$

$$F(\mathbf{x},\mathbf{y},t)_{3M} = \Phi(-\beta(\mathbf{x},\mathbf{y},t)_{3M}) \tag{4.17}$$

where,

$$A = 1+\frac{1}{u_b^2} \tag{4.18}$$

$$u_b = (a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}} - \frac{1}{\alpha_{3G^*}(\mathbf{x},\mathbf{y},t)} \tag{4.19}$$

$$a = -\frac{1}{\alpha_{3G^*}(\mathbf{x},\mathbf{y},t)}\left(\frac{1}{\alpha_{3G^*}^2(\mathbf{x},\mathbf{y},t)} + \frac{1}{2}\right) \tag{4.20}$$

$$b = -\frac{1}{2\alpha_{3G^*}^2(\mathbf{x},\mathbf{y},t)}\sqrt{\alpha_{3G^*}^2(\mathbf{x},\mathbf{y},t) + 4} \tag{4.21}$$

Furthermore, if the first four central moments of the limit state function are available, the fourth-moment reliability index (4M) and the associated estimate of failure probability can be estimated using the high-order moment standardization function.

If z represents a limit state function $z = G(\mathbf{x},\mathbf{y},t)$, the standardized variable z_u can be defined as follows:

$$z_u = \frac{z - \mu_z}{\sigma_z} \tag{4.22}$$

Now, the high-order moment standardization function for standardized variable z_u can be expressed as (Zhao and Ono, 2001):

$$y = z_u - cz_u^2 \text{, and} \tag{4.23}$$

$$u = \frac{y - \mu_y}{\sigma_y} \tag{4.24}$$

where c represents a determinative coefficient.

To make the moments of variable y correspond to the moments of a standard normal variable, the skewness of the distribution for random variable y should be equal to the standard normal variable, or:

$$\alpha_{3y}\sigma_y^3 = 0 \tag{4.25}$$

Finally, by solving Equations 4.23 - 4.25 for the determinative coefficient c, the reliability index and the failure probability can be expressed as:

$$\beta(\mathbf{x},\mathbf{y},t)_{4M} = \frac{3(\alpha_{4G^*}(\mathbf{x},\mathbf{y},t)-1)\beta(\mathbf{x},\mathbf{y},t)_{2M} + \alpha_{3G^*}(\mathbf{x},\mathbf{y},t)(\beta(\mathbf{x},\mathbf{y},t)_{2M}^2 - 1)}{\sqrt{(9\alpha_{4G^*}(\mathbf{x},\mathbf{y},t) - 5\alpha_{3G^*}^2(\mathbf{x},\mathbf{y},t) - 9)(\alpha_{4G^*}(\mathbf{x},\mathbf{y},t)-1)}} \tag{4.26}$$

$$F(\mathbf{x},\mathbf{y},t)_{4M} = \Phi(-\beta(\mathbf{x},\mathbf{y},t)_{4M}) \tag{4.27}$$

Equations 4.15, 4.17, and 4.27, respectively, represent the second moment, the third moment, and the fourth moment cumulative failure probability functions. It is expected that for highly nonlinear limit state functions, the fourth moment cumulative failure probability function would yield the most accurate prediction of failures; this assumption is validated by Damnjanovic and Zhang (2005) for the flexible pavement limit state functions.

With the specified cumulative failure function using the method of moments, the ***reliability function*** $R(\mathbf{x},\mathbf{y},t)$ and the ***hazard rate function*** $h(\mathbf{x},\mathbf{y},t)$ can be respectively determined as follows:

$$R(\mathbf{x},\mathbf{y},t)_{(\cdot)} = 1 - F(\mathbf{x},\mathbf{y},t)_{(\cdot)} = 1 - \Phi\left(-\beta(\mathbf{x},\mathbf{y},t)_{(\cdot)}\right) \tag{4.28}$$

$$h(\mathbf{x},\mathbf{y},t)_{(\cdot)} = -\frac{\partial}{\partial t}\ln[1 - \Phi(-\beta(\mathbf{x},\mathbf{y},t)_{(\cdot)})] \tag{4.29}$$

4.4 Conditional Reliability Functions

Since most of the transportation infrastructure facilities considered for maintenance warranty contracts have been in service, not just recently constructed, the existing reliability needs to be assessed. The conditional reliability is the reliability of a facility that has been in service and survived to the time of the reliability assessment; mathematically, based on the conditional probability theorem, the conditional reliability function can be defined as:

$$R(\mathbf{x},\mathbf{y},t \,|\, A)_{(\cdot)} = \frac{R(\mathbf{x},\mathbf{y},A,t)_{(\cdot)}}{R(\mathbf{x},A)_{(\cdot)}} \tag{4.30}$$

where A represents the estimated accumulated load applications at the beginning of a warranty contract, $R(\mathbf{x},\mathbf{y},t \,|\, A)_{(\cdot)}$ the conditional reliability function, $R(\mathbf{x},\mathbf{y},A,t)_{(\cdot)}$ the joint reliability function, and $R(\mathbf{x},A)_{(\cdot)}$ the probability that the pavement has survived to the time of the reliability assessment.

The limit state function for estimating the reliability at the time of reliability assessment $\left(R(\mathbf{x},A)_{(\cdot)}\right)$ can be defined as:

$$G(\mathbf{x},A) = q(\mathbf{x}) - A \tag{4.31}$$

Equation 4.31 represents the amount of load applications that have not been consumed; or in other words, it represents the remaining capacity of the facility. In general, the variable A can be either a deterministic or a random variable. In this research, without loss of generality, this variable is considered to be deterministic.

Similarly, the joint reliability function $R(\mathbf{x},\mathbf{y},A,t)_{(\cdot)}$ is defined with a limit state function as follows:

$$G(\mathbf{x},\mathbf{y},A,t) = q(\mathbf{x}) - A - z(\mathbf{y},t) \tag{4.32}$$

In contrast to the limit state function defined in Equation 4.31, the limit state function for developing the joint reliability is time-dependent, and it can be interpreted as the remaining capacity at time t from the reliability assessment. Naturally, for defining the limit state functions presented in Equations 4.31 and 4.32, it is necessary to estimate the accumulated load applications at the beginning of a warranty contract (A). In general there are two methods for estimating A: the direct method and the indirect method.

The ***direct method*** for estimating the accumulated load applications at the beginning of a warranty contract is straightforward. If the load applications are measured from the beginning of facility's service life, they can be summed to determine the amount of accumulated load applications at the beginning of a warranty contract; however, such measurements rarely exist.

If the load application measurements from the beginning of the utilization period are not available, the level of accumulated load applications can be estimated from the available condition measurements, such as deflection, roughness, chloride content, carbonation level, or others. In contrast to the straightforward direct method, when traffic count data are available, the condition-based reliability assessment represents an ***indirect method*** for estimating the accumulated load applications at the beginning of a warranty contract.

4.5 Effect of Preventive Maintenance on Reliability Function

Even though reliability models do not explicitly consider the effect of repair and rehabilitations, they still represent a formulation which can be modified to include the effects of preventive maintenance. In contrast to rehabilitations, preventive maintenance actions do not increase the structural capacity, but rather only reduce the rate of deterioration.

To include the effect of preventive maintenance on the reliability function, the reliability function specified in Equation 4.30 can be modified in the following manner (Ebling, 1997):

$$R(\mathbf{x},\mathbf{y},t) = R(\mathbf{x},\mathbf{y},T)^n R(\mathbf{x},\mathbf{y},t-nT) \qquad nT \le t \le (n+1)T \tag{4.33}$$

where T represents the preventive maintenance interval, and n the number of times preventive maintenance action is applied.

Zhang and Piepmeyer (2005) showed that such modification of a reliability function can be effectively used to model the effects of preventive maintenance on pavement deterioration. The underlying assumption of the model specified in Equation 4.33 is that the application of preventive maintenance actions will not increase the reliability; it will just change the rate with which reliability decreases. Since preventive maintenance actions do not increase the reliability, the model is different from a renewal process. Figures 4.2 and 4.3 illustrate the differences in the effects of preventive maintenance and renewal actions on the reliability and the hazard rate functions.

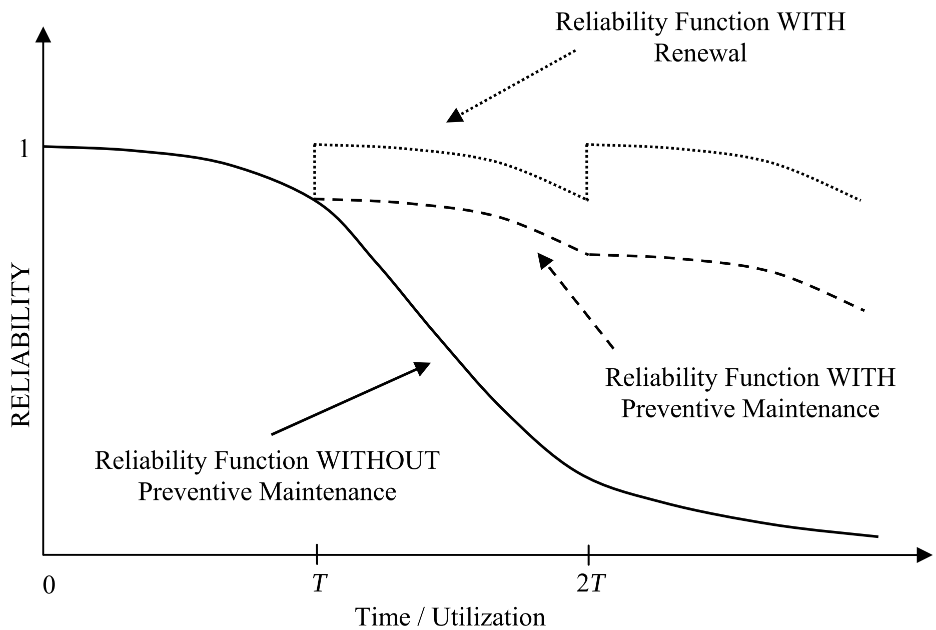


Figure 4.2 Effects of Preventive Maintenance and Renewals on Reliability Function

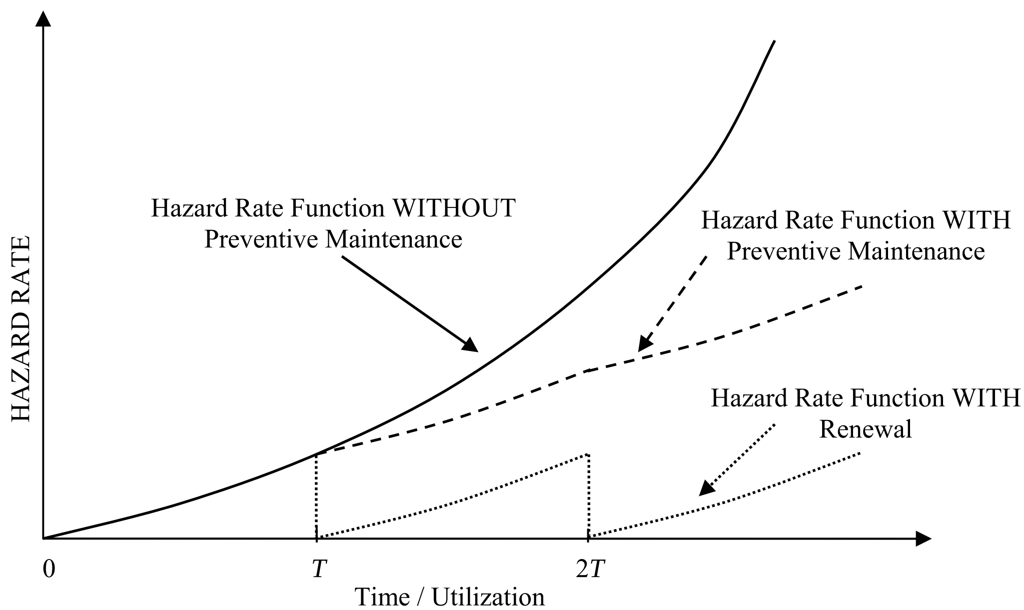


Figure 4.3 Effects of Preventive Maintenance and Renewals on Hazard Rate Function

It can be observed from Figure 4.2 and Figure 4.3 that the effects of preventive maintenance and renewal actions are quite different. While the application of preventive maintenance reduces the deterioration rate through the change in reliability and hazard rate functions, it does not increase the reliability. Many preventive maintenance actions for transportation infrastructure facilities can be classified in this category, such as pavement seal coats, fog seals, and others.

4.6 Summary

This chapter presents a methodology for developing reliability functions based on the method of moments. The time-dependent limit state function is formulated in the first section, while the methodology for evaluating the time-dependent limit state probability integral is presented in the second and the third sections. Finally, in the last two sections, the reliability model is modified to consider: 1) the conditional reliability function, presented in the fourth section, and 2) the effects of preventive maintenance, presented in the fifth section.

In the next chapter, the models that take into account the effects of repairs and rehabilitations are discussed. In addition to the change in the rate of deterioration, these models explicitly consider an increase in the structural capacity.

CHAPTER 5 PERFORMANCE MODELS BASED ON NON-HOMOGENEOUS POISSON PROCESS

This chapter presents the performance models based on the non-homogeneous Poisson process. In contrast to the reliability models discussed in Chapter 4, these models consider the effects of repair and rehabilitations and are applicable for long-term warranty analysis. This chapter consists of two sections. In the first section, a mathematical specification of the model that takes into account only emergency repairs is introduced, while in the second section, the specification of the model is relaxed to include the effects of rehabilitations.

5.1 Model Specification

The non-homogeneous Poisson process (NHPP) is extensively used to describe a system where emergency repair actions are considered. For the NHPP, the effect of emergency repair is minimal; or in other words, after the application of emergency repair, the system is left in an as-bad-as-old condition.

As noted in Chapter 2, the NHPP represents an appropriate representation of the system consisting of many interacting components. From the system point of view, transportation infrastructure facilities are systems composed of many interacting and integrated components. For instance, concrete bridges are composed of several interacting structural elements or subsystems, such as the deck, superstructure, and substructure; similarly, pavements represent a layered structure built over different environments with different subgrade properties.

A stochastic counting process $[N(t), t \geq 0]$ is the NHPP with the ROCOF function $\lambda(t)$ for $t \geq 0$ if:

a) $N(0) = 0$,

b) $[N(t), t \geq 0]$ has independent increments,

c) $\Pr\{N(t+\Delta t) - N(t) \geq 2\} = o(\Delta t)$, the system will not experience more than one failure simultaneously, and

d) $\Pr\{N(t+\Delta t) - N(t) = 1\} = \lambda(t)\Delta t + o(\Delta t)$.

It can be seen from d) that the parameter defining the NHPP is the rate of occurrence of failure (ROCOF) function $\lambda(t)$. For the NHPP, this function is also called peril rate or the failure intensity function. Mathematically, the ROCOF function $\lambda(t)$ can be defined as follows:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\Pr\{N(t+\Delta t) - N(t) \geq 1\}}{\Delta t}, \text{or} \tag{5.1}$$

$$\lambda(t) = \frac{d}{dt} E\left[N(t)\right] \tag{5.2}$$

From Equation 5.2 it can be easily verified that the expected number of failures is equal to the cumulative intensity of the process:

$$E\left[N(t)\right] = \Lambda(t) = \int_0^t \lambda(u)du \tag{5.3}$$

From the definition of the NHPP it can be easily verified that a distribution of the number of failures in the interval $(t_1, t_2]$ is a Poisson distribution (Ross, 1983):

$$\Pr\left\{N(t_2)-N(t_1)=n\right\}=\frac{\left[\Lambda(t_2)-\Lambda(t_1)\right]^n}{n!}e^{-\left[\Lambda(t_2)-\Lambda(t_1)\right]} \quad \text{for} \quad n=0,1,2,...,\infty \qquad (5.4)$$

where $\Lambda(t)=\int_0^t \lambda(t)dt$ represents the cumulative intensity of the process at time *t*, and *n* the number of failures.

Figure 5.1 illustrates the relation between the ROCOF function and the expected number of failures, where the area under the ROCOF function corresponds to the expected number of failures.

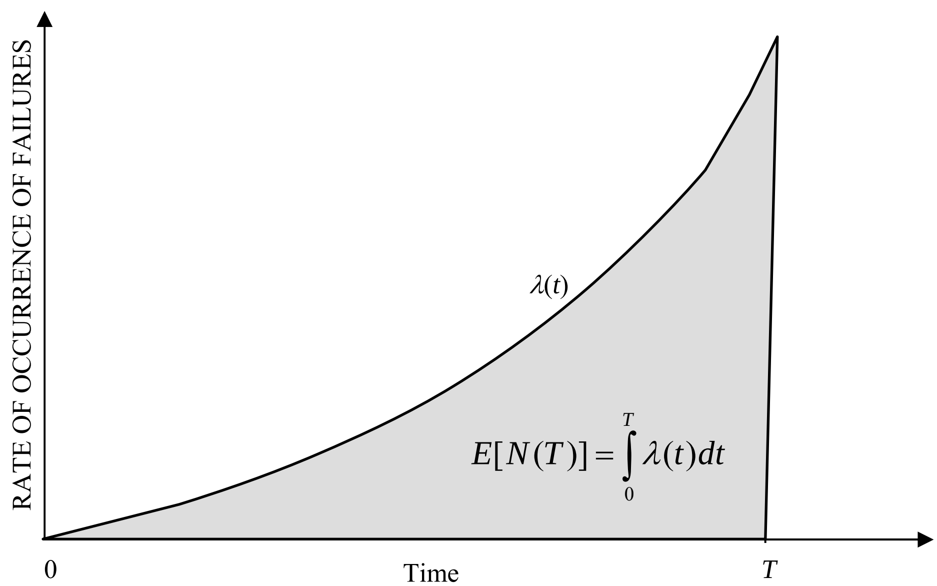


Figure 5.1 Expected Number of Failures for NHPP

5.1.1 Time to First Failure

If T_1 indicates time to the first failure, the survival function can be expressed as follows:

$$S_1(t) = \Pr\{T_1 > t\} = \Pr\{N(t) = 0\} = e^{-\Lambda(t)} = e^{-\int_0^t \lambda(t)dt} \tag{5.5}$$

A consequence of Equation 5.5 is that the hazard rate function of the first inter-arrival time T_1 is equal to the ROCOF function. Hence, if one can estimate the hazard rate function $h(t)$ from reliability theory, then, at the same time, the ROCOF function of the NHPP $\lambda(t)$ can be determined.

However, it is important to understand differences between those two functions. The hazard rate function is used to describe the failure rate of non-repairable systems, while the ROCOF function is used to describe the failure intensity of repairable systems.

5.1.2 Parameters of NHPP and the Method of Moments

As discussed in Chapter 4, the method of moments represents an effective method to evaluate the multidimensional time-dependent probability integral for developing reliability functions. Since the hazard rate function of the first inter-arrival time is equal to the ROCOF function, as noted in Equation 5.5, the method of moments can be also used to develop the ROCOF functions. The relationship between the cumulative hazard rate function and the reliability function is as follows:

$$\mathrm{H}(T) = \int_0^T h(t)dt = -\ln R(T) \tag{5.6}$$

Since the cumulative intensity of the NHPP is equal to the cumulative hazard rate, the following holds:

$$E\left[N(T)\right] = \Lambda(T) = \mathrm{H}(T) \tag{5.7}$$

With the reliability function developed using the limit state function $G(\mathbf{x},\mathbf{y},t)$ and the method of moments used to evaluate the multidimensional time-dependent probability integral, the ROCOF function and the cumulative intensity can be defined respectively as follows:

$$\lambda(\mathbf{x},\mathbf{y},t) = \frac{d}{dt}E\left[N(\mathbf{x},\mathbf{y},t)\right] = \frac{d}{dt}\left[-\ln R(\mathbf{x},\mathbf{y},t)\right], \text{ and} \tag{5.8}$$

$$\Lambda(\mathbf{x},\mathbf{y},t) = -\ln R(\mathbf{x},\mathbf{y},t) \tag{5.9}$$

where $\mathbf{x} \in \mathbb{R}^n$ is a vector of *n* basic random variables in the strength function and $\mathbf{y} \in \mathbb{R}^m$ a vector of *m* basic random variables in the stress function.

5.2 Effects of Rehabilitations on Cumulative Intensity of NHPP

In addition to modeling the effects of preventive maintenance and emergency repairs, performance models for long-term warranty analysis need to account for the effects of rehabilitations. There are many different types of rehabilitations. For example, some pavement rehabilitation measures can result in a pavement structural condition that exceeds the original level; similarly, seismic retrofitting can increase the structural capacity of a bridge to a level greater than the original one. Traditional age-reduction models are unable to account for these effects.

To account for these effects, performance models need to be formulated in a way to explicitly consider the effect of rehabilitations on structural parameters defining the strength in the limit state function. These parameters are also called design variables and represent the variables in the limit state function such that with aging and utilization their values decrease; or in other words, the "strength" of the structure decreases. An example of such a variable is a pavement's structural number (SN): as pavement deteriorates, the SN decreases.

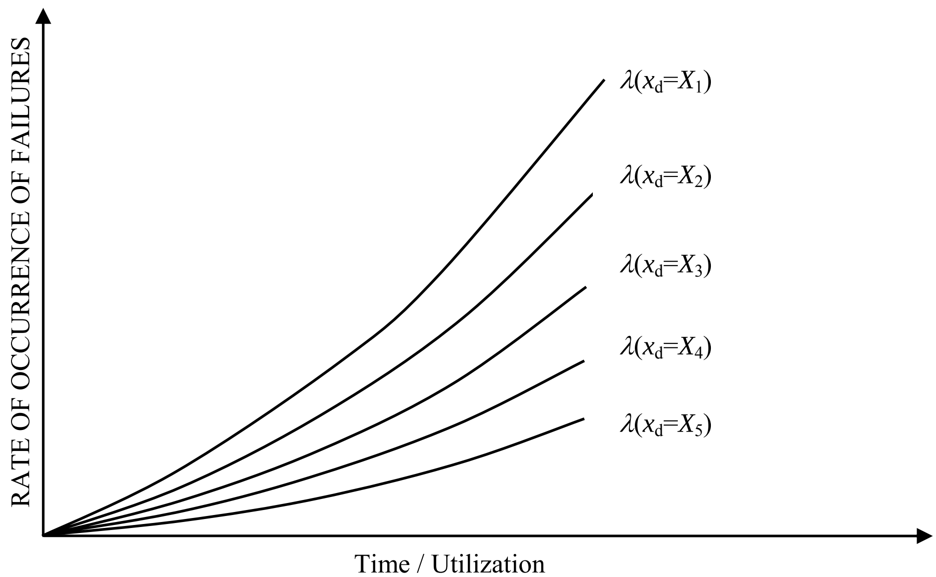
There are two important components of this approach to modeling the effects of rehabilitations: 1) the component for predicting deterioration of design variables in the limit state function, and 2) the component for quantifying the impact of rehabilitations on the level of design variables. The first component describes the deterioration process of design variables, so that the level of design variables just before rehabilitation can be determined. The second component defines an increase in the level of design variables as a result of the application of such rehabilitation.

For example, a recursive function for predicting the level of the design variable (x_d) given the effect of rehabilitation (Δx_d) can be specified as follows:

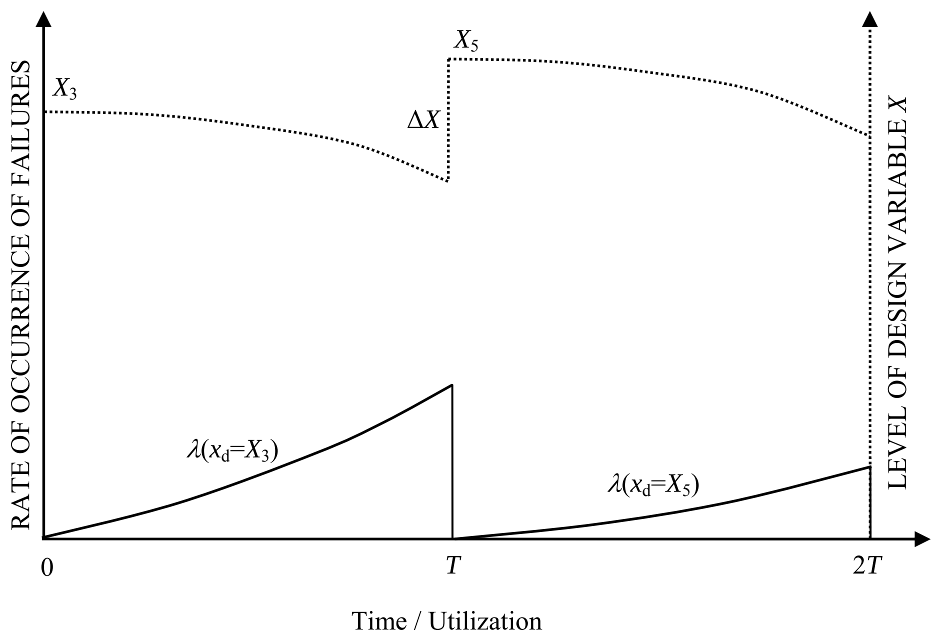
$$x_d(t) = w\left(t, x_d(t-1)\right) + \Delta x_d(t-1) \tag{5.10}$$

where *w* represents a specified deterioration function and $\Delta x_d(t-1)$ an increase in the level of the design variable as a result of applying the rehabilitation at time *t*-1.

After the rehabilitation with $\Delta x_d(t-1)$ effect is taken, the level of the design variable is changed to a new level of $x_d(t)$. With this change from the level of $w\left(t, x_d(t-1)\right)$ to $x_d(t)$, the strength function in the limit state function is updated; consequently, the reliability and the ROCOF functions are also updated to reflect this new level of "strength".



a) ROCOF function for different levels of design variable X



b) ROCOF function for rehabilitation ΔX at time T

Figure 5.2 Effect of Rehabilitations on ROCOF function

The fundamental assumption of this modeling approach is that immediately after the rehabilitation, the probability of failure is zero, or in other words, the reliability equals one. This is a reasonable assumption since it is unlikely that the facility would fail immediately after the rehabilitation is applied, with a condition that the rehabilitations are free of construction blunders. For example, it is highly unlikely that a pavement will fail immediately after an overlay is applied.

Figure 5.2 illustrates this modeling approach, where a) shows the ROCOF function for different levels of the design variable x_d, and b) illustrates the ROCOF function, where the rehabilitation leads to an increase in the level of the design variable x_d to the level of $x_d = X_5$.

As can be observed from part a) of Figure 5.2, the ROCOF function is determined by the initial level of the design variable x_d. As the level of the design variable x_d increases from $x_d = X_1$ to $x_d = X_5$, the ROCOF decreases. If the facility is initially built with the level of the design variable $x_d = X_3$, as illustrated in part b) of Figure 5.2, and if the rehabilitation with the Δx_d effect is applied at time *T*, causing an increase in the level of the design variable to the level of $x_d = X_5$, then, from time *T*, the effective ROCOF function is defined with the level of the design variable $x_d = X_5$.

Mathematically, the expected number of failures during $[0, 2T]$, given the rehabilitation strategy $\Delta x_d(T)$, can be expressed as:

$$E\left[N(2T)\right] = -\ln R(x_d = X_3, \mathbf{x}, \mathbf{y}, T) - \ln R(x_d = X_5, \mathbf{x}, \mathbf{y}, T) \tag{5.11}$$

where x_d represents the design variable, and where

$$x_d(T) = w(x_d = X_3, T-1) + \Delta x_d(T-1) \tag{5.12}$$

5.3 Summary

In this chapter the performance models that account for the effects of emergency repairs and rehabilitations are presented. These models are appropriate for long-term performance warranty analysis and depend on the validity of the NHPP assumption. In the first section, the mathematical specification of the NHPP performance model is presented. The discussion includes: 1) the model specification, 2) the relationship between the cumulative intensity of the stochastic process and the cumulative hazard from reliability theory, and 3) the application of the method of moments to estimate the expected number of failures. In the second section, the transformation of the NHPP model to include the effects of rehabilitations is introduced.

In the following chapter, the risk cost quantification models are presented. The risk cost models are separately formulated for three commonly considered performance warranties: short-term warranties, long-term warranties, and maintenance warranties.

CHAPTER 6 MODELS FOR QUANTIFYING RISK COST

This chapter presents models for quantifying the performance-related risk cost associated with warranties. As noted in Chapter 3, there are three common types of performance warranties: short-term warranties, long-term warranties, and maintenance warranties; consequently, three risk cost quantification models are formulated. This chapter begins with a section discussing the assumptions. In the second and third sections of this chapter, the models for quantification of the risk cost for short-term and long-term warranties are respectively presented. Finally, in the fourth section, the risk cost model for long-term warranty is relaxed to take into account measurements of the accumulated load applications at the beginning of a warranty contract and the condition of the facility at the beginning of a maintenance warranty contract.

6.1 Definitions and Notation

Risk Cost represents the cost incurred during the coverage of performance warranties due to failures. This cost can be defined as the consequence of the failure event times the failure frequency.

Risk Cost Model for Short-term Performance Warranties represents a risk quantification model for short-term performance warranties ranging from two to ten years after construction is completed. Short-term performance warranties do not consider the application of rehabilitations, but often include the application of preventive maintenance; therefore, the performance model needs to account for the effects of preventive maintenance. The performance model that explicitly accounts for the effect of preventive maintenance is specified in Chapter 4.

Risk Cost Model for Long-term Performance Warranties refers to a model for quantifying the risk cost for long-term performance warranties. This type of warranty covers the whole life-cycle of the facility and accounts for the application of both preventive maintenance and rehabilitations. Consequently, the performance of the facility under the coverage of long-term warranties can be modeled using the NHPP assumption. This model assumes minimal emergency repairs, and specifies the effect of rehabilitations directly on the design variables. This performance model is specified in Chapter 5.

Risk Cost Model for Maintenance Performance Warranties represents a type of long-term performance warranties that consider only the exploitation or utilization phase of a transportation infrastructure life-cycle. A unique characteristic of this type of warranty is that at the beginning of a warranty contract, the facility is already in service. Therefore performance models for maintenance warranties need to take into account both measurements of the accumulated load applications at the beginning of a warranty contract and the effect of rehabilitations. Such models are specified in Section 4.4 of Chapter 4 and in Chapter 5.

Preventive Maintenance action is an action that reduces the deterioration rate of the facility. This action does not add additional structural capacity nor increases the reliability; rather, it just reduces the rate of deterioration.

Rehabilitation is an action that increases the structural capacity of the facility; therefore, this action affects the "strength" or the structural capacity part of the limit state function by increasing the allowable level of load applications.

Design Variables are specific variables defining the "strength" function in the limit state function. Not all the variables defining the "strength" of the facility are design variables; only the variables that are under the control of the designer are considered to be design variables.

The following notations are used for developing the risk cost quantification and the warranty-based optimization models.

Indices

t – Time,

p – Preventive maintenance action index,

j – Rehabilitation index,

n – The strength function variable index, and

m – The stress function variable index.

Sets

P – Set of preventive maintenance actions,

J – Set of rehabilitative actions (rehabilitations),

X – Set of variables defining the strength function; where $\mathbf{x} \in \mathrm{X} \in \mathbb{R}^n$ indicates the vector of n basic random variables in the strength function, and

Y	–	Set of variables defining the stress function, where $\mathbf{y} \in \mathrm{Y} \in \mathbb{R}^m$ indicates the vector of *m* basic random variables in the stress function.

Parameters

W	–	Warranty period,
C_F	–	Cost of failure,
C_{PM}	–	Cost of applying preventive maintenance action,
$C\left(\Delta x_d^j\right)$	–	Cost of applying *j*-th rehabilitation that increases the level of design variable for Δx_d^j,
N_{PM}	–	Number of preventive maintenance actions,
T_{PM}	–	Time interval between successive applications of preventive maintenance actions,
i	–	Discount rate,
t_r^j	–	Time between (*j*-1) and *j*-th application of rehabilitation,
Δx_d^j	–	Increase in the level of design variable x_d as a result of *j*-th rehabilitation action, and
A	–	Level of accumulated load application of in-service facility at the beginning of maintenance warranty contract.

Variables

$ERC(W)$	–	Expected risk cost during the warranty period *W*,

$R(t)$ – Reliability at time *t*,

$F(t)$ – Cumulative failure probability at time *t*,

$f_P(t)$ – Probability of failure from time *t*-1 to time *t*,

$\lambda(t)$ – Rate of occurrence of failures function at time *t*,

$VPM(N_{PM}, T_{PM})$ – Value of N_{PM} preventive maintenance actions applied at T_{PM} interval,

$N(W)$ – Number of failures during the warranty period *W*, and

$\theta_{PM}^{j} \in \{0,1\}$ – Binary variable indicating whether the preventive maintenance action has been applied between *j*-1 and *j*-th rehabilitation, when $\theta_{PM}^{j} = 1$; if the preventive maintenance action has not been applied, then $\theta_{PM}^{j} = 0$.

6.2 Short-term Warranties

With the defined consequence of the failure event (C_F), the expected risk cost can be expressed as follows:

$$ERC(\mathbf{x}, x_d, \mathbf{y}, W) = C_F \sum_{t=0}^{W} \frac{1}{(1+i)^t} f_P(\mathbf{x}, x_d, \mathbf{y}, t) \tag{6.1}$$

Since discounting of costs in Equation 6.1 is in discrete time intervals, the failure probability from time *t*-1 to time *t* can be defined as follows:

$$f_P(\mathbf{x}, x_d, \mathbf{y}, t) = F(\mathbf{x}, x_d, \mathbf{y}, t) - F(\mathbf{x}, x_d, \mathbf{y}, t-1) \tag{6.2}$$

In addition, the failure probability can be expressed in terms of the reliability:

$$f_P(\mathbf{x}, x_d, \mathbf{y}, t) = R(\mathbf{x}, x_d, \mathbf{y}, t-1) - R(\mathbf{x}, x_d, \mathbf{y}, t) \tag{6.3}$$

By accounting for the effects of preventive maintenance actions, for $\forall p$, the probability of failure can be reformulated as follows:

$$f_P(\mathbf{x}, x_d, \mathbf{y}, t) = R(\mathbf{x}, x_d, \mathbf{y}, T_{PM})^p \left[R(\mathbf{x}, x_d, \mathbf{y}, t-1-pT_{PM}) - R(\mathbf{x}, x_d, \mathbf{y}, t-pT_{PM}) \right] \tag{6.4}$$

Finally, based on the failure probability specified in Equation 6.4, the expected risk cost can be estimated as:

$$\begin{aligned} ERC(\mathbf{x}, x_d, \mathbf{y}, W) = C_F \sum_{p=0}^{N_{PM}} \sum_{t=pT_{PM}}^{\min\{W,(p+1)T_{PM}\}} \frac{R(\mathbf{x}, x_d, \mathbf{y}, T_{PM})^p}{(1+i)^t} \big[R(\mathbf{x}, x_d, \mathbf{y}, t-1-pT_{PM}) - \\ - R(\mathbf{x}, x_d, \mathbf{y}, t-pT_{PM}) \big] \end{aligned} \tag{6.5}$$

where N_{PM} represents the number of preventive maintenance applications, or the greatest integer less than or equal to the length of the warranty period *W* divided by the preventive maintenance interval T_{PM}.

In Equation 6.5, the outer summation is over the number of preventive maintenance intervals, while the inner summation is over the time between two consecutive preventive maintenance actions; to account for the interval from the last application of preventive maintenance to the end of the warranty period (*W*), the inner summation limit is specified as a minimum of either the length of the warranty period (*W*) or the last application of preventive maintenance. Since the preventive maintenance is applied in equal intervals, depending on the actual length of the warranty period (*W*), the number of applied preventive maintenance actions is determined as the greatest integer less than or equal to the length of the warranty period *W* divided by the length of preventive maintenance interval T_{PM}.

The risk cost quantification model, presented in Equation 6.5, can also be used to calculate the financial benefits of applying preventive maintenance actions for short-term warranty projects. The value of preventive maintenance actions can be estimated by comparing different preventive maintenance scenarios with a default scenario where no preventive maintenance action is applied. Mathematically, this can be specified as:

$$VPM(N_{PM}, T_{PM}, W) = ERC(N_{PM} = 0, T_{PM} = 0, W) - ERC(N_{PM}, T_{PM}, W) \tag{6.6}$$

The preventive maintenance action that maximizes the difference between the value of preventive maintenance and the cost of applying preventive maintenance is the optimal policy for the contractor servicing the warranty. In Chapter 7, the risk cost quantification model, presented in this section, is reformulated as the warranty-based optimal design problem for design-build projects.

6.3 Long-term Warranty

For long-term warranties, the expected risk cost without considering the effects of preventive maintenance and rehabilitations can be expressed as a product of the cost of failure and the expected number of failures, or mathematically:

$$ERC(\mathbf{x}, x_d, \mathbf{y}, W) = C_F E\left[N(\mathbf{x}, x_d, \mathbf{y}, W)\right] = -C_F\left[\ln R(\mathbf{x}, x_d, \mathbf{y}, W)\right] \tag{6.7}$$

With a preventive maintenance action applied in equal intervals, the risk cost quantification model can be formulated as follows:

$$\begin{aligned} ERC(\mathbf{x}, x_d, \mathbf{y}, W) = -C_F \Bigg[& (1+\theta_{PM}) \ln R\left(\mathbf{x}, x_d, \mathbf{y}, \frac{W}{(1+\theta_{PM})}\right) + \\ & + \frac{W\theta_{PM}}{(1+\theta_{PM})} \lambda\left(\mathbf{x}, x_d, \mathbf{y}, \frac{W}{(1+\theta_{PM})}\right) \Bigg] \end{aligned} \tag{6.8}$$

When the preventive maintenance action is not applied, $\theta_{PM} = 0$, and Equation 6.8 becomes equivalent to Equation 6.7. To account for the effects of rehabilitations, the model can be further expanded. For example, the expected risk cost model that considers rehabilitations can be specified as a model comprising of the sum of the two risk cost expectations: 1) the expected risk cost incurred in the period from the beginning of the warranty contract to the application of the last rehabilitation (ERC_I), and 2) the expected risk cost incurred from the application of the last rehabilitation to the end of the warranty coverage (ERC_{II}):

$$ERC(\mathbf{x}, x_d, \mathbf{y}, W) = ERC_I + ERC_{II} \tag{6.9}$$

where the expected risk cost components are defined as follows:

$$\begin{aligned} ERC_I = -C_F \Bigg\{ \sum_{j=1}^{J} \Bigg[(1+\theta_{PM}^j) \ln R\left(\mathbf{x}, x_d\left(\sum_{i=1}^{j} t_r^i \right), \mathbf{y}, \frac{t_r^j}{(1+\theta_{PM}^j)} \right) + \\ + \frac{t_r^j \theta_{PM}^j}{(1+\theta_{PM}^j)} \lambda \left(\mathbf{x}, x_d\left(\sum_{i=1}^{j} t_r^i \right), \mathbf{y}, \frac{t_r^j}{(1+\theta_{PM}^j)} \right) \Bigg] \Bigg\} \end{aligned} \tag{6.10}$$

$$\begin{aligned} ERC_{II} = -C_F \Bigg\{ (1+\theta_{PM}^{J+1}) \ln R\left(\mathbf{x}, x_d\left(W - \sum_{j=1}^{J} t_r^j \right), \mathbf{y}, \frac{W - \sum_{j=1}^{J} t_r^j}{(1+\theta_{PM}^{J+1})} \right) + \\ + \frac{\left(W - \sum_{j=1}^{J} t_r^j \right) \theta_{PM}^{J+1}}{(1+\theta_{PM}^{J+1})} \lambda \left(\mathbf{x}, x_d\left(W - \sum_{j=1}^{J} t_r^j \right), \mathbf{y}, \frac{\left(W - \sum_{j=1}^{J} t_r^j \right)}{(1+\theta_{PM}^{J+1})} \right) \Bigg\} \end{aligned} \tag{6.11}$$

As can be observed, the risk cost quantification model, presented in Equation 6.9, consists of two parts: 1) the expected number of failures given the application of J

rehabilitations; this expectation is represented with a summation of the expected number of failures over the members of the set of rehabilitations J, and 2) the expected number of failures in the last interval of the warranty coverage, from the last rehabilitation to the end of the coverage of warranty policy, or in the interval $W - \sum_{j=1}^{J} t_r^j$.

In general, depending on the type of transportation infrastructure and the corresponding performance indicator, a deterioration function of the design variable can be defined in many different functional forms; for example, the level of a design variable at time *t* after the application of the *j*-th rehabilitation can be specified as:

$$x_d^j(t) = w\left(t, x_d(t=0), \Delta x_d^1, \ldots, \Delta x_d^{j-1}\right) + \Delta x_d^j \qquad \forall j \qquad (6.12)$$

The value of applying specific preventive maintenance and rehabilitation strategies can be evaluated in a similar manner as for short-term warranties: by comparing an estimate of the risk cost when no preventive maintenance and rehabilitations are taken, with an estimate of the risk cost for specific preventive maintenance and rehabilitation strategies. In Chapter 7, the risk cost quantification model, presented in this section, is reformulated as the warranty-based optimal design problem for design-build projects.

6.4 Maintenance Warranty

The expected risk cost model for a facility that has been in service but has not received preventive maintenance or rehabilitation during the coverage of warranty, can be expressed as follows:

$$ERC(\mathbf{x}, x_d, \mathbf{y}, W) = C_F E\left[N(\mathbf{x}, x_d, \mathbf{y}, W)\middle| A\right] = -C_F\left[\ln R(\mathbf{x}, x_d, \mathbf{y}, W\middle| A)\right] \tag{6.13}$$

where $\ln R(\mathbf{x}, x_d, \mathbf{y}, W|A)$ represents the conditional expectation given that the facility has survived A amount of load applications.

To account for preventive maintenance, rehabilitation, and the level of accumulated load applications at the beginning of a warranty contract, the risk cost model needs to consider three risk costs: 1) the expected risk cost given the level of accumulated load application at the beginning of a warranty contract, from the beginning of contact to the first rehabilitation (ERC_I); 2) the expected risk cost from the first rehabilitation to the last rehabilitation (ERC_{II}); and 3) the expected risk cost from the last rehabilitation to the end of the warranty coverage period (ERC_{III}). Mathematically, this can be specified as follows:

$$ERC(\mathbf{x}, x_d, \mathbf{y}, W) = ERC_I + ERC_{II} + ERC_{III} \tag{6.14}$$

where the expected risk cost components can be defined as follows:

$$\begin{aligned} ERC_I = -C_F \Bigg\{ & (1+\theta_{PM}^1)\ln R\left(\mathbf{x}, x_d\left(t=0\right), \mathbf{y}, \frac{t_r^1}{(1+\theta_{PM}^1)}\middle| A\right) + \\ & + \frac{t_r^1\theta_{PM}^1}{(1+\theta_{PM}^1)}\lambda\left(\mathbf{x}, x_d\left(t=0\right), \mathbf{y}, \frac{t_r^1}{(1+\theta_{PM}^1)}\middle| A\right)\Bigg\} \end{aligned} \tag{6.15}$$

$$\begin{aligned} ERC_{II} = -C_F \Bigg\{ \sum_{j=1}^{J}\Bigg[& (1+\theta_{PM}^j)\ln R\left(\mathbf{x}, x_d\left(\sum_{i=1}^{j} t_r^i\right), \mathbf{y}, \frac{t_r^j}{(1+\theta_{PM}^j)}\right) + \\ & + \frac{t_r^j\theta_{PM}^j}{(1+\theta_{PM}^j)}\lambda\left(\mathbf{x}, x_d\left(\sum_{i=1}^{j} t_r^i\right), \mathbf{y}, \frac{t_r^j}{(1+\theta_{PM}^j)}\right)\Bigg]\Bigg\} \end{aligned} \tag{6.16}$$

$$ERC_{III} = -C_F \left\{ (1+\theta_{PM}^{J+1}) \ln R\left(\mathbf{x}, x_d \left(W - \sum_{j=1}^{J} t_r^j \right), \mathbf{y}, \frac{W - \sum_{j=1}^{J} t_r^j}{(1+\theta_{PM}^{J+1})} \right) + \right.$$

$$\left. + \frac{\left(W - \sum_{j=1}^{J} t_r^j \right) \theta_{PM}^{J+1}}{(1+\theta_{PM}^{J+1})} \lambda \left(\mathbf{x}, x_d \left(W - \sum_{j=1}^{J} t_r^j \right), \mathbf{y}, \frac{\left(W - \sum_{j=1}^{J} t_r^j \right)}{(1+\theta_{PM}^{J+1})} \right) \right\} \tag{6.17}$$

Similar to the long-term warranty risk cost model, the deterioration model for the design variable can be specified with Equation 6.12, and the value of preventive maintenance and rehabilitations can be determined using Equation 6.6. In Chapter 7, the risk cost quantification model, presented in this section, is reformulated as an optimization problem for optimal scheduling of rehabilitations for maintenance warranty contracts.

6.5 Summary

This chapter presents the models for quantifying the expected risk cost for performance warranty contracts. The first section introduces the definition and notation, while the second, third, and fourth sections, respectively, present the risk cost models for short-term warranties, long-term warranties, and maintenance warranties. In the next chapter, the risk cost quantification models are reformulated as warranty-based optimization models.

CHAPTER 7 OPTIMAL WARRANTY-BASED DESIGN AND MAINTENANCE SCHEDULING MODELS

This chapter presents optimization models for determining the optimal design and maintenance scheduling policy. In contrast to the commonly considered life-cycle cost optimization models, these models include the risk cost or the warranty servicing cost. Often, the optimization models that include such cost components are referred to as optimal warranty-based models. In the first section, the formulation of the optimal warranty-based design and maintenance scheduling models are presented and discussed. Since convexity can not be guaranteed, the solution algorithm for non-convex problems is presented in the second section. In the third section, a general formulation for sensitivity analysis for the warranty-based optimization problems is introduced. Finally, in the fourth section, the basic features of the models are summarized.

7.1 Problem Definition

While optimization models have been extensively used for determining optimal schedule of maintenance actions for transportation infrastructure, most of the applications were focused either on network-level analysis, or on traditional life-cycle cost analysis. To account for the specific features of warranty-based contracting, the optimization models need to consider the expected risk cost or the warranty servicing cost.

In this chapter, the optimization models for three types of performance warranties are presented: the optimal design model for short-term warranties, the integrated optimal design and maintenance scheduling model for long-term warranties, and the optimal maintenance scheduling model for maintenance warranties.

The following assumptions are made in the process of developing warranty-based optimization models:

1. Without loss of generality, the design variable x_d is a scalar and is considered a decision variable in the optimization model;
2. The design variable x_d can not be perfectly controlled in the construction process and is therefore considered to be a random variable;
3. The construction cost is a continuous and increasing function of the design variable, or mathematically, $C(x_d) = f_C(x_d)$;
4. If it is considered, the preventive maintenance schedule is predetermined and conducted in equal intervals; and
5. The contractor can purchase new equipment, or improve the construction process in order to reduce the variability of the design variable.

7.1.1 Short-term Warranties

The warranty-based optimal design problem for short-term warranties can be defined as determining the level of the design variable that minimizes the total expected cost. In addition to the risk cost, or the warranty servicing cost, the two other cost components contributing to the total cost are preventive maintenance cost and as-designed initial construction cost.

Therefore, the objective of this model is to minimize this total expected cost by determining the level of the design (decision) variable x_d. Since the reliability function based on the method of moments is a nonlinear function of the design and the other basic random variables in the limit state function, the optimal warranty-based design problem can be formulated as a nonlinear optimization model.

Keeping the same notations as defined at the first section of Chapter 6, the ***objective*** can be expressed as follows:

$$\begin{aligned} &\min_{x_d} C(x_d) + C_{PM} + ERC(\mathbf{x}, x_d, \mathbf{y}, W) \\ &s.t. \;\; x_d^{lower} \le x_d \le x_d^{upper} \end{aligned} \tag{7.1}$$

where $ERC(\mathbf{x}, x_d, \mathbf{y}, W)$ represents the expected risk cost given the level of the design variable x_d.

If the validity of the strength function and the limit state function is constrained over some domain of design (decision) variables, a constraint that sets a lower and upper bound on the level of decision variables can be introduced. To reflect local requirements, additional constraints can be included to the model presented in Equation 7.1, such as minimum reliability levels.

This problem has a unique solution only if the objective function specified in Equation 7.1 is convex over the considered interval of the design variable x_d. In general, convexity of the objective function, as defined in Equation 7.1, depends on the nonlinearities of the limit state function as well as the nonlinearities introduced in the process of estimating the reliability function using the method of moments. Therefore, it cannot be claimed that the design optimization problem is a convex problem for an arbitrarily defined limit state function. However, even though the objective function might not be convex for the entire domain of design variables, it still might be convex for the values in a feasible domain, defined by the validity of the limit state function.

For all practical purposes, such problems are considered to be convex problems and can be easily solved using standard nonlinear search algorithms, such as the steepest descent or the Newton-Raphson generalized algorithm. The algorithm for solving the problem defined in Equation 7.1 is shown in Figure 7.1 (Nash and Sofer, 1996).

initialization
  Setup Optimality Criterion (ε)
  Setup Iteration Limit (N_L)
Compute $\nabla f(x^{(0)})$
if $\nabla f(x^{(0)}) = [0+\varepsilon]$ **then**
  $x^{(0)}$ is optimal solution – **STOP**
end if
$x^{(i)} = x^{(0)}$
do while $\nabla f(x^{(*)}) = [0+\varepsilon]$
  Compute a search direction $d(x^{(i)})$ using the gradient $\nabla f(x^{(i)})$.
  Determine the step size distance $s_{(i)}$ which is an approximation of the value that minimizes the objective function $f\left(x^{(i)} + s^{(i)} d(x^{(i)})\right)$.
  $x^{(i+1)} = x^{(i)} + s^{(i)} d(x^{(i)})$
end do

Figure 7.1 Pseudocode for General Descent Algorithm

Since the two cost components contributing to the total cost are dependent on the level of the design variable x_d, the optimization problem can be illustrated as shown in Figure 7.2. It can be observed from Figure 7.2 that as the mean level of the decision variable x_d increases, the expected risk cost during the warranty period decreases; however, with such an increase in the mean of the design (decision) variable x_d, the construction cost increases. Therefore, the optimization problem is to find the level of the decision variable that yields the minimum total cost.

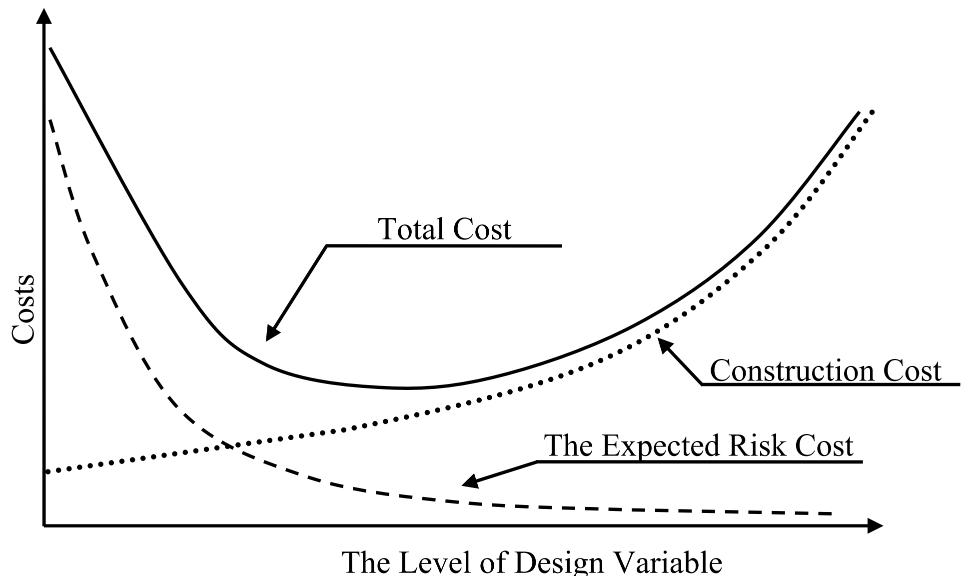


Figure 7.2 Relationship Between Level of Design Variable and Cost Components

The optimization problem presented in this section can also be interpreted as a stochastic program with simple recourse. Once the first-stage decision is made, in terms of the mean level of the design variable, the uncertainty is resolved and the warranty servicing cost is determined. Since short-term warranties do not consider the application of rehabilitations, this model does not include a second-stage recourse action. In fact, this aspect of the problem distinguishes the simple recourse model presented in this section from the recourse model for the long-term warranty optimization presented in the next section.

7.1.2 Long-term Warranties

The warranty-based optimal design problem for long-term warranties can be specified as follows: determine the level of the design variable and the rehabilitation strategy that minimizes the total cost comprising of the risk cost, or the warranty servicing cost, preventive maintenance cost, rehabilitation cost, and as-designed initial construction cost. Therefore, for long-term warranty analysis, the total expected cost includes all the life-cycle cost components.

The following assumptions are made for formulating the integrated warranty-based design and maintenance scheduling optimization model:

1. The NHPP and the ROCOF function are defined using the method of moments and applied to model the performance of the facility during the warranty period;
2. The effects of rehabilitations are quantified using the model that takes into account the effect of rehabilitations on the design variable. A deterioration model for determining the effective level of the design variable, introduced in Chapter 6, is adapted for developing the optimization model;
3. After the rehabilitation is applied, the ROCOF function is defined based on the effective (current) level of the design variable; and
4. In addition to the design variable (x_d), the time to *j*-th rehabilitation (t_r^j) and the intensity of *j*-th rehabilitation (Δx_d^j) are also continuous variables. These variables are also decision variables.

There are two types of decision variables that the optimization model for long-term warranties considers: 1) continuous variables, such as the previously mentioned design variable (x_d), time to the *j*-th rehabilitation (t_r^j), and the intensity of the *j*-th rehabilitation (Δx_d^j); and 2) an integer variable *j*, or the number of rehabilitations considered during the warranty period. Therefore, the original optimization problem is a mixed-integer nonlinear program, a class of optimization problems that are computationally difficult.

To overcome this difficulty, the problem can be reformulated as a two-step optimization process. First, the optimal design strategy is determined for the pre-defined number of rehabilitations; this sub-problem is a nonlinear problem where convexity is not guaranteed. Second, after the minimal total cost for *J* scenarios of rehabilitations is determined, the overall optimal strategy is determined as a strategy with the minimum total cost.

Keeping the same notation as defined in the first section of Chapter 6, the ***objective*** for a rehabilitation scenario, where *j* rehabilitations are considered, can be expressed as follows:

$$\min_{x_d, \mathbf{\Delta x_d^j}, \mathbf{t_r^j}} C(x_d) + C(\mathbf{\Delta x_d^j}) + N_{PM} C_{PM} + ERC(\mathbf{x}, x_d, \mathbf{\Delta x_d^j}, \mathbf{t_r^j}, \mathbf{y}, W) \qquad \forall j \tag{7.2}$$

where $C(\mathbf{\Delta x_d^j})$ indicates the cost function for *j* rehabilitations with intensities Δx_d^j, and $ERC(\mathbf{x}, x_d, \mathbf{\Delta x_d^j}, \mathbf{t_r^j}, \mathbf{y}, W)$ the expected risk cost as defined in Chapter 6.

The overall optimal strategy can be found as:

$$TEC^* = \min\left\{TEC\left(x_d^{\ *}, \Delta x_d^{1^*}, t_r^{1^*}\right), \ldots, TEC\left(x_d^{\ *}, \Delta x_d^{1^*}, \ldots, \Delta x_d^{J^*}, t_r^{1^*}, \ldots, t_r^{J^*}\right)\right\} \tag{7.3}$$

This optimal design strategy includes the initial level of the design variable as well as the rehabilitation strategy, where the optimal rehabilitation strategy can consider one to J rehabilitations during the warranty period. In contrast to commonly considered life-cycle models that separate design and maintenance optimization, and therefore yield a sub-optimal solution, the model presented in Equation 7.2 considers both optimization problems simultaneously.

As is the case with the short-term warranty-based optimization model, the optimization problem for the long-term warranties is also not heavily constrained. Mainly, the constraints of this problem are the validity of the strength function and the minimal level of rehabilitations. Still, the solution to this problem is not easily obtainable due to fact that the problem is non-convex. The solution approach to this not-heavily-constrained non-convex problem is presented in Section 7.2.

7.1.3 Maintenance Warranties

The warranty-based optimal maintenance scheduling problem for maintenance warranties can be defined as follows: determine a rehabilitation strategy that minimizes the total expected cost comprising the risk cost, or the warranty servicing cost, preventive maintenance cost, and rehabilitation cost. Therefore, the optimization problem for maintenance warranties represents a special case of the long-term warranty problem, where the initial as-designed construction cost is not considered.

In addition to the assumptions made for developing the long-term warranty optimization model, the assumption used for determining the optimal rehabilitation strategy for maintenance warranties is that the effects of previous utilization at the beginning of the warranty contract can be assessed. This can be done either through measurements of load applications (direct method), or by associating the level of structural capacity and distress indicators with the accumulated number of load applications (indirect method).

Keeping the same notations as defined at the first section of Chapter 6, the objective is to minimize the total costs comprising: 1) the cost of the rehabilitation strategy, 2) the cost of preventive maintenance, and 3) the warranty servicing cost. For *j* number of rehabilitations, the ***objective*** can be expressed as follows:

$$\min_{\boldsymbol{\Delta}\mathbf{x}_\mathbf{d}^j, \mathbf{t}_\mathbf{r}^j} C(\boldsymbol{\Delta}\mathbf{x}_\mathbf{d}^j) + N_{PM} C_{PM} + ERC(\mathbf{x}, x_d, \boldsymbol{\Delta}\mathbf{x}_\mathbf{d}^j, \mathbf{t}_\mathbf{r}^j, \mathbf{y}, W) \qquad \forall j \tag{7.4}$$

where $C(\boldsymbol{\Delta}\mathbf{x}_\mathbf{d}^j)$ indicates the cost function for *j* rehabilitations with intensities Δx_d^j, and $ERC(\mathbf{x}, x_d, \boldsymbol{\Delta}\mathbf{x}_\mathbf{d}^j, \mathbf{t}_\mathbf{r}^j, \mathbf{y}, W)$ the expected risk cost as defined in Chapter 6.

Similar to the long-term warranty optimization model, the maintenance optimization model can be solved using the proposed two-step process. For *J* scenarios of rehabilitation policy, the overall optimal rehabilitation strategy can be found as:

$$TEC^* = \min\left\{TEC\left(\Delta x_d^{1^*}, t_r^{1^*}\right), \ldots, TEC\left(\Delta x_d^{1^*}, \ldots, \Delta x_d^{J^*}, t_r^{1^*}, \ldots, t_r^{J^*}\right)\right\} \tag{7.5}$$

Since the maintenance optimization model represents just a special case of the long-term optimization model, this model is also not heavily constrained and is generally considered to be non-convex. In the following section, the solution approach to non-convex problems is presented.

7.2 Mixed Multi-start Solution Algorithm for Non-Convex Warranty-Based Design and Rehabilitation Scheduling Problems

As previously discussed, the long-term warranty-based design and the maintenance scheduling problems are not-heavily-constrained non-convex problems. The main constraint in these problems is a bound on the level of decision variables.

Over the years, non-convex problems, or problems where convexity is not guaranteed, have attracted the attention of researchers. The most common solution approach to these problems is the application of heuristic methods, such as the genetic algorithm, tabu search, and scatter search. Even though these multi-start methods are capable of avoiding the trap of local minima, the quality of the solution is often poor; in fact, the first derivative at the minimum point from the heuristic solution is often quite different than zero. In other words, the candidate solution from the heuristic solution is not the minimizer in the local convex region. To overcome this problem, Ugray et al. (2002) proposed an algorithm that avoids local minimizers by employing a robust scatter search heuristic that converges to the minimizer by utilizing standard nonlinear search algorithms, such as Newton-Raphson generalized algorithm. This mixed multi-start scatter search nonlinear programming algorithm is composed of two subroutines: the Scatter search subroutine (Glover, 1998) and the NLP subroutine (Nash and Sofer, 1996).

initialization

Size of *Solution Set*

Size of *Reference Set*

Start $P = \emptyset$

while $card\left|P\right| < Size\ of\ \ Solution\ Set$ **do**

Use the "*diversification generation method*" to construct the solution and apply the "*improvement method*". Let x be solution.

if $x \notin P$ **then**

$P = P \cup x$

end if

end while

Use the "*reference set update method*" to build $Reference\ Set = \left\{x_1,\ldots,x_b\right\}$ with best solutions in *P*.

Order solutions in $Reference\ Set$ such that x_1 is the best and x_b is the worst.

Make $New\ Solutions = TRUE$

while ($New\ Solutions$) **do**

Generate $New\ Subsets$ with the subset generation method.

Make $New\ Solutions = FALSE$

while ($New\ Subsets = \emptyset$) **do**

Select the next subset *s* in $New\ Subsets$

Apply the "*solution combination method*" to get trail solutions.

Apply the "*improvement method*" to trail solutions

Apply the "*reference set update method*"

if ($Reference\ Set$ has changed) **then**

Make $New\ Solutions = TRUE$

end if

Delete *s* from $New\ Subsets$

end while

end while

Figure 7.3 Scatter Search Subroutine Pseudocode

The scatter search subroutine (Glover, 1998) is presented in Figure 7.3, while the pseudocode for the NLP subroutine used in the mixed algorithm for non-convex problems (Nash and Sofer, 1996) is illustrated in Figure 7.4. Since the global minimum is assumed to occur in a feasible domain defined by the lower and upper bounds on decision variables, the mixed multi-start solution algorithm first finds a population of the candidate start points using the scatter search method, then it passes the candidate solution to the NLP algorithm to find the minimizer in the local convex region (Ugray et al., 2002). This procedure is illustrated in Figure 7.5.

---

initialization

Read Model Parameters
Setup Optimality Criterion
Setup Iteration Limit

Compute $\nabla f(x_{NLP}^{(0)})$

if $\nabla f(x_{NLP}^{(0)}) = [0+\varepsilon]$ **then**

$x_{NLP}^{(0)}$ is optimal solution – STOP

end if

$x_{NPS}^{(i)} = x_{NPS}^{(0)}$

do while $\nabla f(x_{NLP}^{(*)}) = [0+\varepsilon]$

Compute a search direction $d(x_{NLP}^{(i)})$ using the gradient $\nabla f(x_{NLP}^{(i)})$.

Determine the search distance $s_{(i)}$ which is an approximation of the value that minimizes the objective function $f\left(x_{NPS}^{(i)} + s_{(i)} d(x_{NPS}^{(i)})\right)$.

$x_{NPS}^{(i+1)} = x_{NPS}^{(i)} + s_{(i)} d(x_{NPS}^{(i)})$

end do

Figure 7.4 Nonlinear Programming (NLP) Subroutine Pseudocode

initialization

Read *Model* Parameters
Setup Nonlinear Programming Subroutine (*NPS*) Parameters
Setup Scatter Search Subroutine (*SSS*) Parameters
Stage 1 *Iteration Limit* or Number of calls of *SSS*
Size of the SSS list
Maximum Distance Factor
Stage 1 *Iteration = 0*

<u>*STAGE* 1</u>:

while (*Stage 1 Iteration* < *Stage 1 Iteration Limit*) **do**

Solve *Model* using *SSS*

$$\left\{ f^*_{sss}(Stage\ 1\ Iteration),\ x^*_{sss}(Stage\ 1\ Iteration),\ L^* = \bigcup_{1}^{i} L^*(Stage\ 1\ Iteration = i) \right\}$$

if ($f^*_{sss}(Stage\ 1\ Iteration) \le f^*_{sss}$) **then**

$L^* = L^* \cup \left\{ x^*_{sss} \right\}$

$f^*_{sss} = f^*_{sss}(Stage\ 1\ Iteration)$

$x^*_{sss} = x^*_{sss}(Stage\ 1\ Iteration)$

end if

end while

<u>*STAGE* 2</u>:

Let $x^*_{sss} = x^{(0)}_{NLP}$

Solve *Model* using *NPS*

$\left\{ f^*_{NPS},\ x^*_{NPS}, d^* = \left| x^{(0)}_{NPS} - x^*_{NPS} \right| \right\}$

while ($L^* = \emptyset$) **do**

if $d^*_l = \left| x^{(0)}_l - x^*_{NPS} \right| \le maximum\ distance\ \ factor \times d^*$ **then**

Solve *Model* using *NPS*

$\left\{ f^*_{l,NPS},\ x^*_{l,NPS}, d^*_l = \left| x^{(0)}_L - x^*_{NPS} \right| \right\}$

if ($f^*_{l,NPS} \le f^*_{NPS}$) **then**

$f^*_{NPS} = f^*_{l,NPS}$

$x^*_{NPS} = x^*_{l,NPS}$

$d^* = d^*_l$

end if

end if

Delete *l* from L^*

end while

Figure 7.5 Mixed Multi-Start Algorithm Pseudocode

7.3 Sensitivity Analysis

In addition to convexity, another problem often considered in reliability-based optimization is sensitivity analysis (Grierson, 1983). The objective of sensitivity analysis is to establish a measure of how the objective function varies with changes in the parameters of the problem.

The Lagrangian formulation of the problem represents a straightforward approach to sensitivity analysis (Castillo et al., 2004). First, a non-constrained problem is transferred to the equivalent constrained problem. This is done by declaring the parameters of the optimization model to be decision variables, and then locking them by the means of constraints. For example, if $x^{'}$ and $y^{'}$ are the parameters of interest for sensitivity analysis of the short-term warranty optimization model, the problem can be transferred to the equivalent form as follows:

$$\min_{x_d} C(x_d) + C_{PM} + ERC(\mathbf{x}, x_d, \mathbf{y}, W) \tag{7.6}$$

subjected to,

$$x^{'} = \overline{x}$$

$$y^{'} = \overline{y}$$

With such a formulation, the problem becomes a nonlinear problem with linear equality constraints. To solve this problem, the problem can be reformulated using its dual formulation. In fact, every nonlinear problem has an associated dual problem. The dual problem of the primal problem, defined in Equation 7.6, can be defined as follows:

$$\max_{\lambda,\mu} \operatorname{Inf}_{\lambda,\mu,x_d,x^{'},y^{'}} \left\{ \mathfrak{A}(\lambda, \mu, x_d, x^{'}, y^{'}) \right\} \tag{7.7}$$

where the Lagrangian function $\mathfrak{A}(\bullet)$ is defined as:

$$\mathfrak{A}(\lambda, \mu, x_d, x^{'}, y^{'}) = C(x_d) + C_{PM} + ERC(\mathbf{x}, x_d, \mathbf{y}, W) + \lambda(x^{'} - \overline{x}) + \mu(y^{'} - \overline{y}) \qquad (7.8)$$

Then, the sensitivity of the objective function with the respect to the basic random variables $\overline{x}$ and $\overline{y}$ can be assessed through the values of the dual variables (Lagrange multipliers) λ^* and μ^*.

7.4 Summary

This chapter presents the models for determining the optimal warranty-based design and maintenance schedule. In the first section, the formulation of the optimal warranty-based design and maintenance scheduling models are introduced, while in the second section, a mixed multi-start solution algorithm for non-convex problems is presented. In the third section, a general formulation for the sensitivity analysis for the warranty-based optimization problems is discussed. In the following chapter, a case study is presented to illustrate the methodology.

CHAPTER 8 CASE STUDY

This chapter presents a case study to test the accuracy of the method of moments and to illustrate the overall methodology. The case study uses pavements as an example of transportation infrastructure. In the first section, the current AASHTO method for design of pavements is presented. In the second section, the accuracy of the method of moments to estimate the failure probability is tested. In the third, fourth, and fifth section of this chapter, the risk cost quantification and the warranty-based optimization models are respectively presented for short-term, long-term, and maintenance warranties.

8.1 AASHTO Design Method

One of the most widely-used methods for designing pavements is the AASHTO design method. This design procedure is based on the results from the accelerated pavement testing experiment, known as the AASHO Road Test (AASHTO, 1993).

Without considering the term that accounts for the overall variance, the strength of a flexible pavement defined by the AASHTO design equation is given as follows:

$$\begin{aligned}\log W_{18} &= 9.36\log(SN+1) - 0.20 + \frac{\log[\Delta PSI/(4.2-1.5)]}{0.4+1094/(SN+1)^{5.19}} + \\ &\quad + 2.32\log M_r - 8.07\end{aligned} \tag{8.1}$$

where W_{18} represents the allowable number of equivalent 18-kip (80-kN) single-axle loads (ESAL) to cause a reduction of the serviceability level by the amount of ΔPSI, SN the structural number, and M_r the effective resilient modulus of roadbed soil.

Similarly, without considering the term that considers the overall variance, the strength of a rigid pavement, defined by the modified AASHTO design equation, is given as follows:

$$\log W_{18} = 7.35\log(D+1) - 0.06 + \frac{\log[\Delta PSI/(4.5-1.5)]}{1+1.624\times10^{7}/(D+1)^{8.46}} + \\ +(4.22-0.32p_t)\left\{\frac{S_c C_d (D^{0.75}-1.132)}{215.63J[D^{0.75}-18.42/(E_c/k)^{0.25}]}\right\} \tag{8.2}$$

where W_{18} represents the allowable number of equivalent 18-kip (80-kN) single-axle loads (ESAL) to cause a reduction of the serviceability level by the amount of ΔPSI, D the slab thickness in inches, S_c the modulus of rupture of concrete, E_c the modulus of elasticity of concrete, k the modulus of subgrade reaction, J the load transfer coefficient, C_d the drainage coefficient, and p_t the terminal serviceability.

With the established pavement strength using the AASHTO design equations, to formulate the limit state function, the stress must be considered in the same units as the strength. The time-dependent stress can be defined as the accumulated number of equivalent 18-kip (80-kN) single-axle loads (ESAL).

There are many different methods to predict future traffic in ESALs. Regardless of the method used, two major parts of the traffic analysis can be identified. First, the initial yearly traffic in ESALs is estimated by transforming various load groups into equivalent 18-kip ESALs with corrections for the effects of lane and direction distributions. Second, the traffic growth is estimated with different functional forms of the growth factor. Both the Asphalt Institute (Huang, 1993) and the AASHTO pavement design guide (AASHTO, 1993) recommend the use of a growth factor that considers traffic over the entire design period:

$$TGF(r,t) = \frac{(1+r)^t - 1}{r} \tag{8.3}$$

where *TGF* represents the total growth factor, *r* the yearly rate of growth, and *t* the design period. With a modified yearly traffic in ESALs for the initial year ($ESAL_0$) and the total growth factor *TGF*, the stress or the accumulated ESALs for an arbitrary time period *t* can be obtained as:

$$N(ESAL_0, r, t) = ESAL_0 \times TGF(r,t) = ESAL_0 \times \frac{(1+r)^t - 1}{r} \tag{8.4}$$

Given that the stress is defined by Equation 8.4 and the strength by either Equation 8.1 or Equation 8.2, the limit state function can be expressed as:

$$G(\cdot,t) = \log W_{18} - \log N(t) \tag{8.5}$$

The limit state function $G(\cdot,t)$ and the failure domain, defined as $\{G(\cdot,t) \le 0\}$, establish the time-dependent probability integral:

$$F(\cdot,t) = \int_{G(\cdot,t)\le 0} f(\cdot,t)\,d(\cdot) \tag{8.6}$$

To find the higher-order moments of the limit state function for flexible pavements, the limit state function can be approximated in an equivalent linear form according to the procedure given by Zhao and Ono (2001):

$$G^*(SN, M_r, ESAL_0, r, t) = G_{SN} + G_{M_r} + G_{ESAL_0} + G_r - 3G_\mu \tag{8.7}$$

Similarly, the limit state function for rigid pavements can be approximated as follows:

$$G^*(D, S_c, E_c, k, ESAL_0, r, t) = G_D + G_{S_c} + G_{E_c} + G_k + G_{ESAL_0} + G_r - 5G_\mu \tag{8.8}$$

The main advantage of this approximation is the ability to address the computational difficulty associated with point estimation in the standard normal space. In Equations 8.7 and 8.8, $G_{(\cdot)}$ represents the functions in which all the basic random variables are evaluated using their mean values, except for the basic random variable that appear in the index of the function. Finally, the central moments of the limit state functions and the reliability indices can be estimated as discussed in Chapter 4, where the cumulative failure, the reliability, and the hazard rate function can be expressed as follows:

$$F(\cdot,t) = \Phi\left(-\beta(\cdot,t)\right) \tag{8.9}$$

$$R(\cdot,t) = 1 - F(\cdot,t) = 1 - \Phi\left(-\beta(\cdot,t)\right) \tag{8.10}$$

$$\lambda(\cdot,t) = -\frac{\partial}{\partial t}\ln[1 - \Phi(-\beta(\cdot,t))] \tag{8.11}$$

8.2 Accuracy of the Method of Moments

In order to test the applicability of the method of moments to estimate the failure probabilities, a comparison analysis was conducted between the estimates of the failure probabilities from Monte Carlo simulation (MCS) and the method of moments.

Considering that the basic random variables in the limit state function are normally distributed, Monte Carlo simulations and three different methods of moments were used to estimate the failure probability of the pavement from the initial construction to 50 years after the pavement was put in service. Also, the basic random variables in the limit state function are assumed to be independent.

Table 8.1 shows the means and the coefficients of variation of the basic random variables for a flexible pavement's limit state function, while Table 8.2 shows the means and the coefficients of variation of the basic random variables for a rigid pavement's limit state function. The values for the coefficient of variation were taken from previous studies (Huang, 1993).

Table 8.1 Parameters of Basic Random Variable Used in Comparison Analysis for Flexible Pavements

Random Basic Variable	Mean	Coefficient of Variation
Modified Yearly ESAL	109261	10%
Resilient Subgrade Modulus	7000	15%
Structural Number	3.61	10%
Yearly ESAL Growth Rate	0.08	10%

Table 8.2 Parameters of Basic Random Variable Used in Comparison Analysis for Rigid Pavements

Basic Random Variable	Mean	Coefficient of Variation
Slab Thickness	8.2	4%
Modulus of Rupture	650	10%
Modulus of Elasticity	5,000,000	5%
Modulus of Subgrade Reaction	72	35%
Yearly ESAL Growth Rate	0.08	10%
Modified Yearly ESAL	109,261	10%

For each time period, the failure probability was estimated with a Monte Carlo simulation of 1 million samples or a total of 50 million samples for the entire period. Because of the large sample size, the estimates of failure probabilities from MCS are considered to be the actual, or the true failure probabilities; therefore the accuracy of the method of moments is assessed as a difference between the failure probability estimates from MCS and the method of moments. The results from the comparison study indicated the expected trend; the higher the order of the moments in the reliability index, the smaller the estimation error.

Figure 8.1 illustrates the difference between the estimated failure probabilities for flexible pavements using MCS and those using the methods of moments for the first five years. For this time period, the 3M method overestimates the failure probability by 16 percent, while the 2M method underestimates the failure probability by over 43 percent. It is important to note that the prediction errors are even greater if shorter time periods are considered. In contrast to the 2M and 3M methods, the 4M method generally produces estimates with better accuracy. According to the results, the 4M method underestimates the failure probability for the first five years by only four percent.

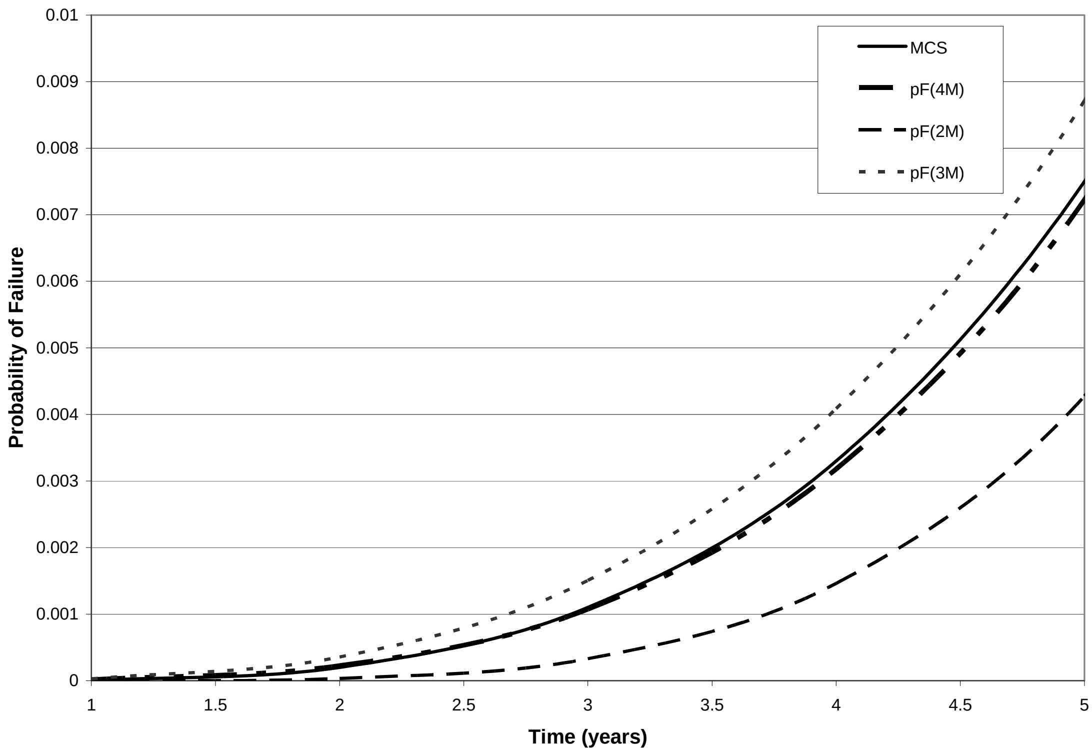


Figure 8.1 Comparison of Failure Probabilities Using Methods of Moments and MCS

Table 8.3 shows the mean and the maximum absolute errors for the estimated failure probabilities, where the corresponding reliability levels are for the periods in which the maximum absolute error occurs. In general, there is no specific trend indicating that one method always underestimates or overestimates the failure probability; rather, the results from the analysis indicate that the method of moments produces estimates that oscillate around the actual failure probability values. For example, even though the 3M method overestimates the failure probability for the first five years, after year 12, it starts to underestimate the failure probability. The maximum absolute error produced by the 4M method is the smallest compared to the 2M and 3M methods. A similar trend can be observed if the mean absolute error is considered; the

4M method yields an error of 0.0001 compared with the 3M and 2M methods that give errors of 0.0026 and 0.0047, respectively.

Table 8.3 Summary of Errors in Prediction of Failure Probability for Flexible Pavements

Method	Maximum Absolute Error	Mean Absolute Error
2M Reliability Index	0.01330	0.00468
3M Reliability index	0.00765	0.00262
4M Reliability Index	0.00060	0.00012

In addition, the accuracy of the method of moments was assessed for rigid pavements. Figure 8.2 illustrates the difference between the estimated failure probabilities using MCS and the methods of moments from year 4 to year 7. Similar to flexible pavements, the trend of obtaining better estimates of the failure probabilities with the higher-order moments is also observed for rigid pavements.

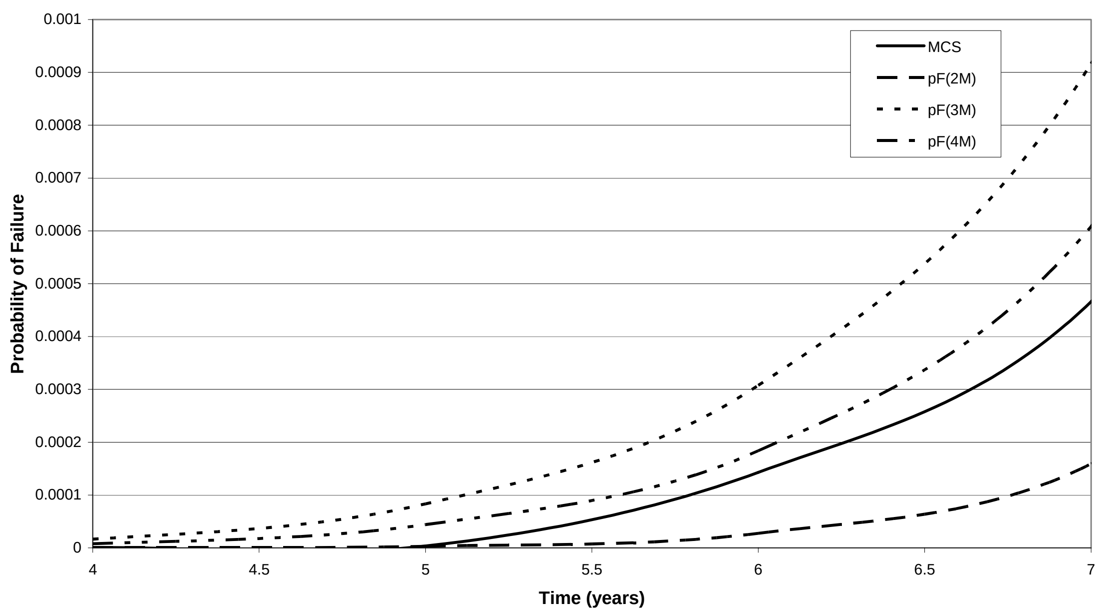


Figure 8.2 Comparison of Failure Probabilities Using Methods of Moments and MCS

Table 8.4 summarizes the mean and the maximum absolute errors for the estimated failure probabilities for rigid pavements. It can be seen from Table 8.4 that the maximum and the mean absolute errors produced by the 4M method are smallest when compared to the 2M and 3M methods. In percentages, the mean error from the 4M method for the design period of 20 years is approximately 10 percent.

Table 8.4 Summary of Errors in Prediction of Failure Probability for Rigid Pavements

Method	Maximum Absolute Error	Mean Absolute Error
2M Reliability Index	0.01599	0.00272
3M Reliability index	0.00957	0.00231
4M Reliability Index	0.00632	0.00099

Overall, for both flexible and rigid pavements, the results show that the method of moments can accurately estimate the failure probability and can be used for developing pavement reliability functions. Even though the limit state function was highly nonlinear and therefore not normally distributed, the comparison results between MCS and the method of moments are encouraging. The results indicate that the same methodology can be used for assessing the reliability for different design approaches. To further illustrate the methodology, the case study is expanded to include the development of the risk cost quantification and the warranty-based optimization models for flexible pavements.

8.3 Short-term Warranties

For both the risk cost quantification model and the warranty-based design model, Equation 8.10 represents the pavement performance model given the variability of the basic random variables used in the limit state function, including the design variable *SN*.

Therefore, using the AASHTO design equations for flexible pavements, the risk cost model can be formulated as follows:

$$ERC(SN,W) = C_F \sum_{n=0}^{N} \sum_{t=nT}^{\min\{W,(n+1)T\}} \frac{R(T,SN)^n}{(1+i)^t} \left[R(t-1-nT,SN) - R(t-nT,SN) \right] \quad (8.12)$$

Assuming that the initial as-designed construction cost increases linearly with an increase in the level of design variable *SN*, the warranty-based design optimization model can be formulated as follows:

$$\min_{SN} TEC(SN,W) = a \times SN + ERC(SN,W) \quad (8.13)$$

subjected to:

$SN_{\min} \le SN \le SN_{\max}$

8.3.1 Numerical Example

This section presents a numerical example to illustrate the behavior of the developed model. The data used in the analysis are summarized in Table 8.1, while the discount rate *i* is assumed to be 5 percent. Without loss of generality, these random variables are assumed to be normal and uncorrelated.

Since the failure event was common for all scenarios $(\Delta PSI = 2)$, the warranty policy is defined by considering different values of the other three indicators of the warranty policy: the failure cost, ranging from \$100,000 to \$400,000 per lane per mile; the length of the warranty period, from 7 to 13 years; and the application of preventive maintenance, applied at year 5 after the construction, or omitted. The consideration of preventive maintenance allows both the SHA and contractors to quantify the financial benefit of applying preventive maintenance; or in other words, it allows for estimating the value of preventive maintenance for warranty projects. The costs are reported in dollars per lane per mile.

Figure 8.3 illustrates the estimates of the risk cost for different scenarios of a warranty policy, where the failure cost was fixed to $200,000 per lane per mile.

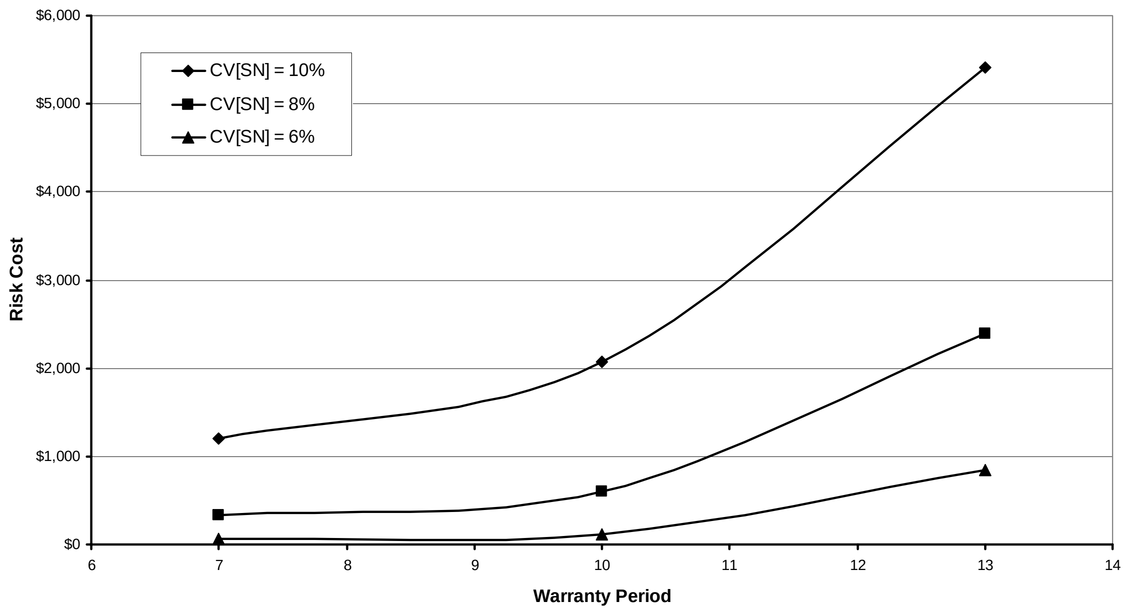


Figure 8.3 Estimated Risk Costs Given Different Warranty Periods and CV[SN]

It can be observed from Figure 8.3 that a decrease in variability of the design variable *SN* can significantly reduce the risk cost. This observation reinforces the argument that with the application of performance warranties, the contractors will be motivated to implement much stricter quality control measures.

In addition to estimating the risk cost, the model provides estimates of the value of preventive maintenance actions. Figure 8.4 shows the value of preventive maintenance for different lengths of the warranty policy. In this numerical example, as intuitively expected, the application of preventive maintenance becomes a more attractive option as the failure cost, the coefficient of variation of the *SN*, and the length of the warranty period increases.

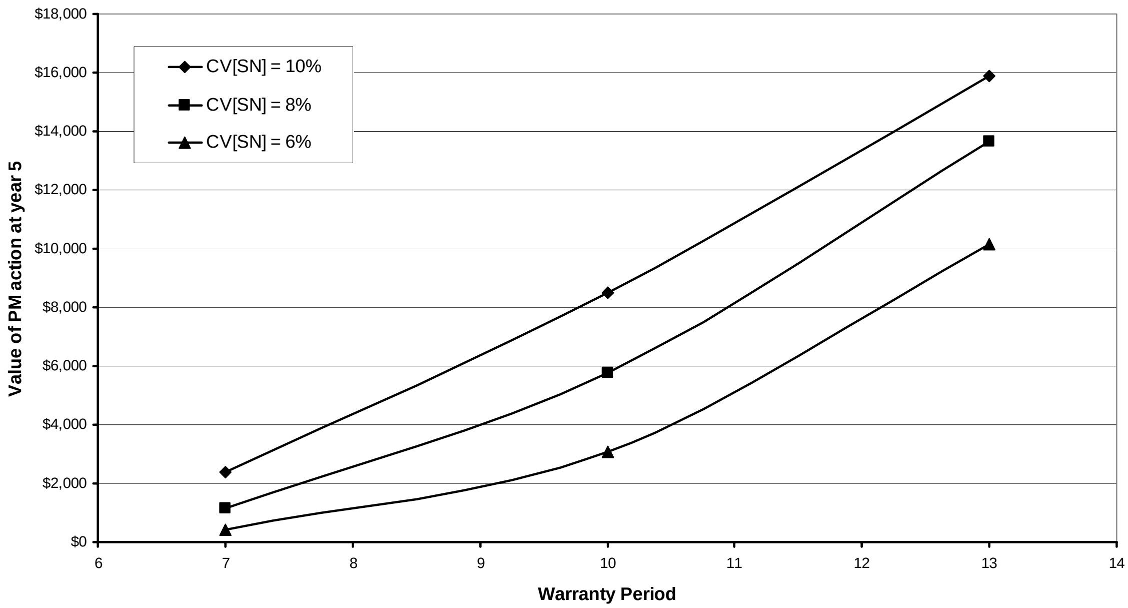


Figure 8.4 Value of Preventive Maintenance for Different CV[SN]

In the warranty-based design optimization model, the level of the *SN* is not predetermined from the AASHTO design equation; rather it is solved for by minimizing the objective function defined as the total expected costs incurred during the warranty period. Furthermore, the parameter of the initial as-designed construction cost *a*, as defined in the objective function, is considered to be $20,000 per lane per mile. In the numerical example for the design optimization model, the warranty scenarios and the values assigned to the random variables are identical to those used for estimating the risk cost.

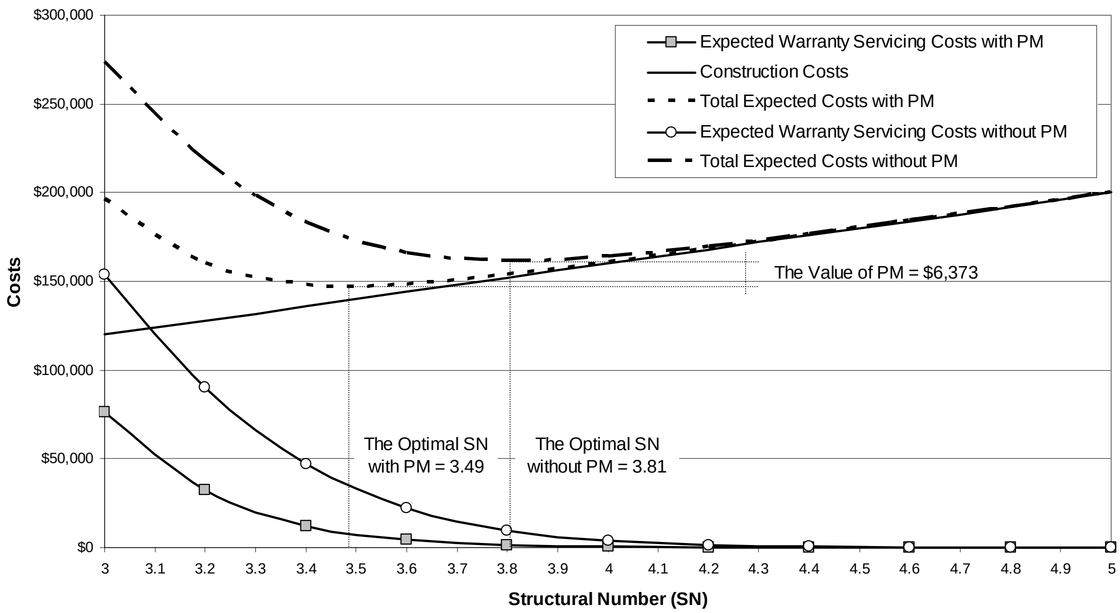


Figure 8.5 Breakeven of Cost Contribution to the Optimization Model: With and Without Application of PM at Year 5

Figure 8.5 shows the breakeven of the cost components in the optimization model, where the failure cost was $400,000 and the warranty period 10 years. It can be seen from the figure that the problem is convex over the *SN* interval from 3 to 5. This represents a condition that the solution is a unique and optimal solution. Furthermore, Figure 8.5 illustrates two different warranty policy scenarios, with or without the application of preventive maintenance. It can be observed that the optimal level of the design variable *SN* decreases when preventive maintenance is considered. It changes from *SN* = 3.81, without the application preventive maintenance, to *SN* = 3.49 with the consideration of a preventive maintenance at year 5. Therefore, with the specified warranty policy, the value of preventive maintenance to contractors is approximately

$6,400 per lane per mile. This means that if the cost of preventive maintenance exceeds this value, the contractors would be better off without it.

Finally, Figure 8.6 shows the sensitivity of the optimal *SN* to changes in the coefficient of variation of the *SN*, and the unit *SN* to failure cost ratio. It can be observed from Figure 8.6 that as the level of *SN* coefficient of variation decreases, the optimal *SN* also decreases. This indicates that a reduction in variability of the design variables can lead to the design of a thinner pavement that can withstand the same traffic loading.

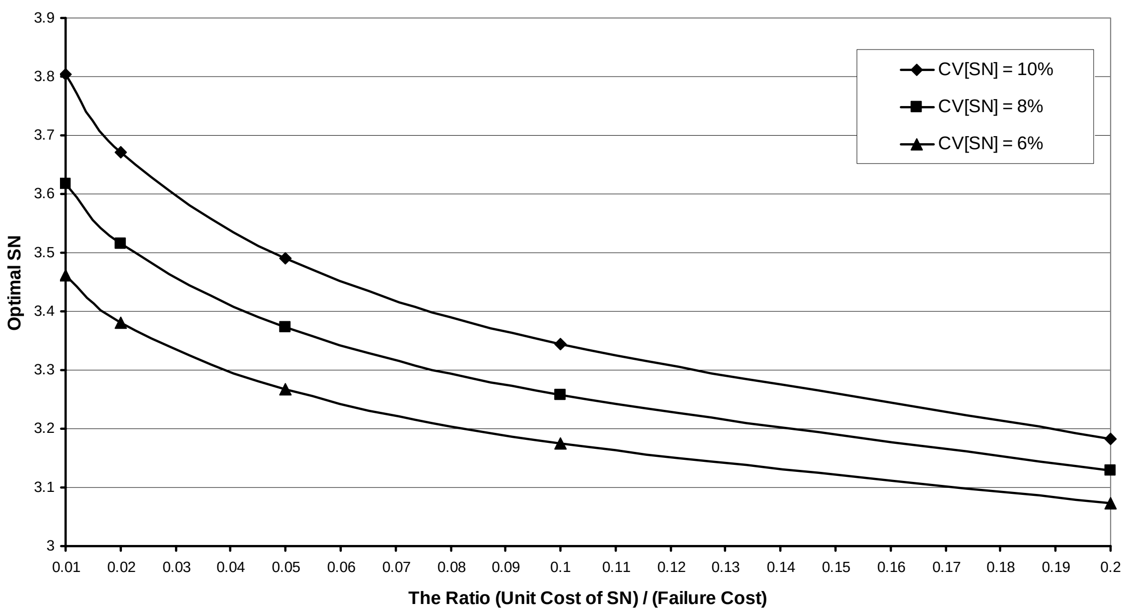


Figure 8.6 Changes in Optimal *SN* with Changes in Cost Ratio and CV[*SN*]

Furthermore, the financial benefit of an investment in new equipment, or an improvement in the construction processes, aimed at reducing of the variability of *SN*, can be assessed by comparing the total costs for the optimal *SN* at different levels of its coefficient of variation. For example, in the case when the ratio is 0.01, by reducing the coefficient of variation of *SN* from 10 percent to 6 percent, the optimal *SN* decreases from

3.8 to less than 3.5. In general, the higher the cost of failure, the more efficient it is to reduce total costs by reducing the level of the *SN* coefficient of variation, rather than to design a thicker pavement.

8.4 Long-term Warranties

As discussed in Chapter 5, the NHPP model can be specified with the limit state function. Using the AASHTO design equations, the limit state function for analysis of long-term warranties is identical to the formulation of the limit state function for short-term warranties. With the reliability function developed using the method of moments, the expected number of failures in the warranty period can be expressed as:

$$E\left[N(SN,W)\right]=\int_0^W \lambda(SN,u)du=-\ln R(SN,W) \tag{8.14}$$

Equation 8.14 represents the performance model given the initial level of the design variable *SN*, where the expected number of failures represents the performance measure. Then, the expected risk cost (*ERC*) for long-term warranties, where the AASHTO design equations are used to define the limit state function, can be specified as presented in Chapter 6 and Equations 6.9 and 6.10. Finally, the objective function of the warranty-based design optimization model, considering a scenario with *j* rehabilitations, can be defined as:

$$\min_{SN,\boldsymbol{\Delta} SN^j,\mathbf{t}_{\Delta SN^j}} C(SN)+C(\boldsymbol{\Delta} SN^j)+ERC(\cdot,SN,\boldsymbol{\Delta} SN^j,\mathbf{t}_{\Delta SN^j},W) \tag{8.15}$$

The optimal solution of this problem can be found by comparing the minima for *J* rehabilitation scenarios. To be consistent with the AASHTO design method, the effective

SN can be defined using the condition index and the remaining life concept. The condition index, or the ratio between the effective and the initial *SN* can be defined as follows (AASHTO, 1993):

$$C_x = \frac{SN_{eff}}{SN} = 1 - 0.7 \times \exp[-(R_L + 0.85)^2] \tag{8.16}$$

where the remaining life is $R_L = \dfrac{W_{18} - N(t)}{W_{18}}$.

The assumption of Equation 8.16 is that the *SN* of a pavement would never be completely consumed. Based on the condition index function, at the time of failure, the effective *SN* of a pavement is reduced to approximately two-thirds of the initial value.

Based on Equation 8.16, the effective *SN* at time *t*, after the rehabilitation is taken (increasing the *SN* by ΔSN), can be expressed as:

$$SN_{eff}(t) = SN\left[1 - 0.7 \times \exp\left[-\left(\frac{W_{18} - N(t)}{W_{18}} + 0.85\right)^2\right]\right] + \Delta SN \tag{8.17}$$

8.4.1 Numerical Example

This section presents a numerical example to illustrate the behavior of the developed models. The data used in the analysis are summarized in Table 8.1. In addition, preventive maintenance is not considered, and the minimum level of rehabilitations is set to be 0.45, which corresponds approximately to a one-inch overlay. The analysis is performed for a 4-lane and 5-mile highway section, where the length of the warranty period is 20 years and the failure cost varies. Table 8.5 summarizes the minimum total costs for different rehabilitation scenarios.

Table 8.5 Summary of Results from Numerical Example

Number of Rehabilitaion Actions	Total costs	Initial SN
0	$2,215,554	4.13
1*	$2,117,347	3.57
2	$2,228,459	3.36
3	$2,340,536	3.17

It can also be observed from Table 8.5 that with an increase in the number of rehabilitations, the initial optimal level of *SN* decreases. Such behavior is expected and intuitive since an option to consider rehabilitation later in the service life provides an incentive to build a weaker pavement to start, and then apply rehabilitation as the traffic is realized. The optimal solution problem is found when only one rehabilitation is applied during the warranty period, with the initial *SN* value of 3.57.

In addition to building stronger pavements to transfer the performance-related risk, the contractors can also choose to reduce variability of the design variable *SN* to reduce such costs. Figure 8.7 shows the effect of a reduction in the coefficient of variation of the *SN* on the minimum total costs, while Figure 8.8 illustrates the effects of the ratio of the failure to unit *SN* costs on the optimal level of the initial *SN*.

It can be observed from Figure 8.7 that there is almost a linear trend in the cost reduction due to the reduction of variability of the design variable *SN*. On the other hand, as illustrated in Figure 8.8, the effect of the previously defined ratio is quite different from linear. In fact, there is a nonlinear relationship between the initial optimal *SN* and the failure to unit *SN* cost ratio. An increase in this ratio has a greater effect on the initial optimal *SN* when the ratio changes from 2 to 3 than from 3 to 4. Again, this behavior is anticipated: as the failure cost increases, it becomes more cost efficient to build thicker pavements in order to avoid paying higher failure costs.

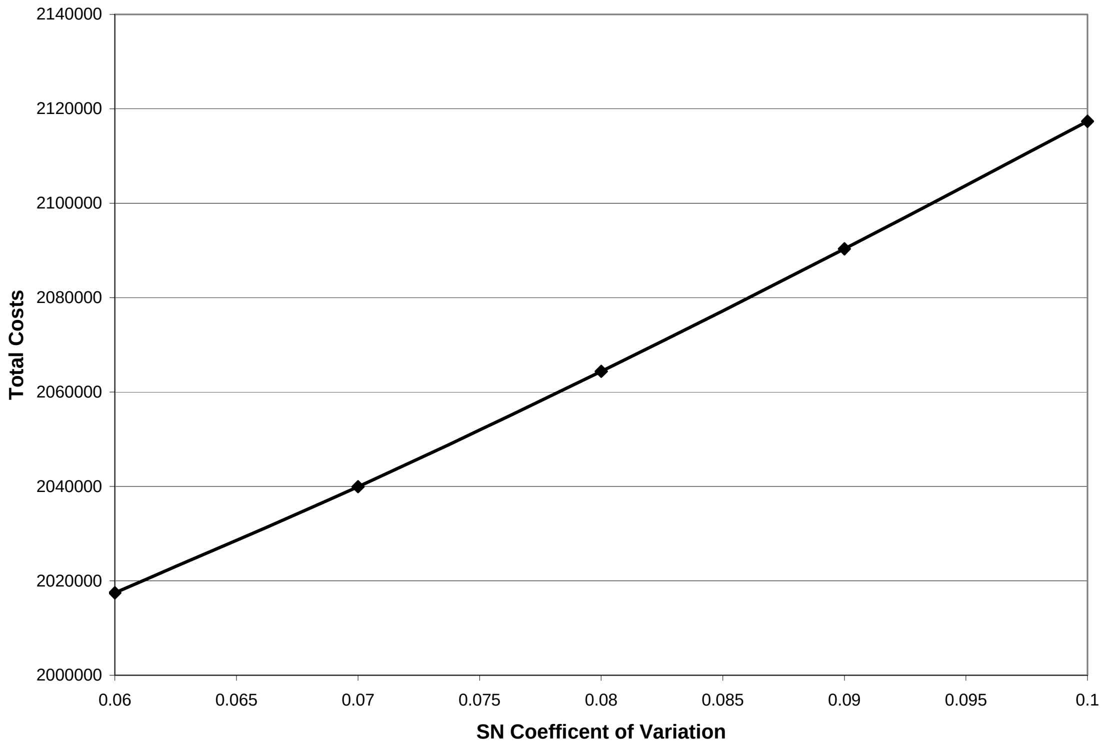


Figure 8.7 Effect of Reduction of Variability on the Total Costs

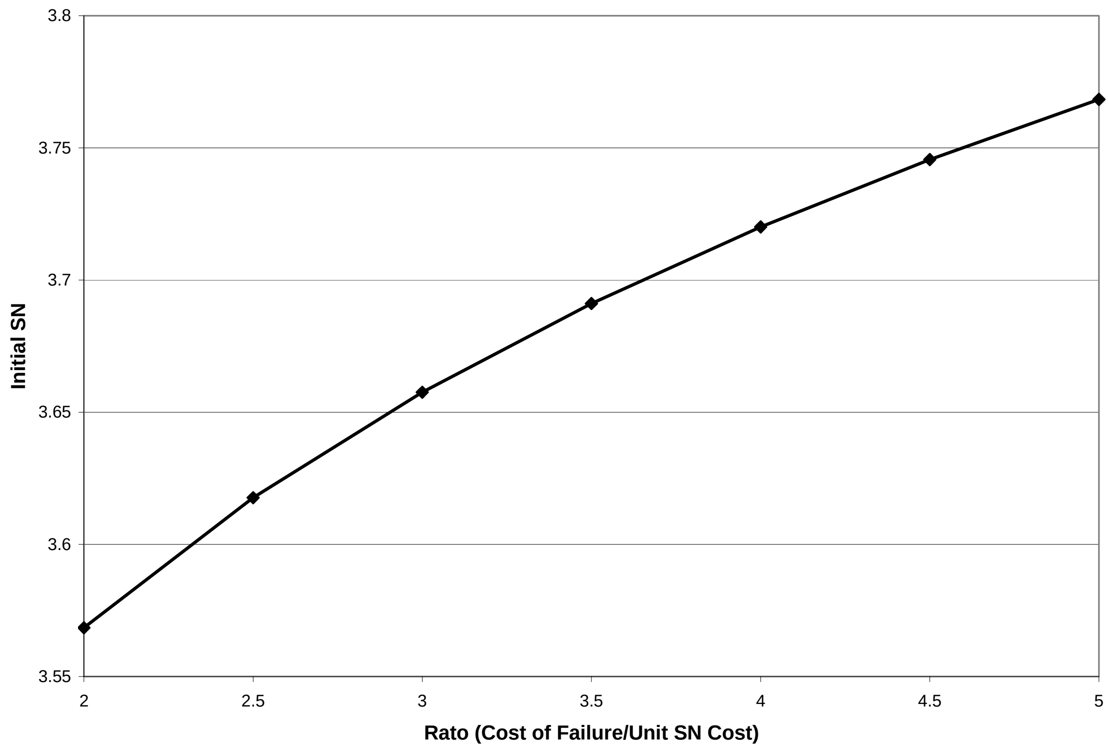


Figure 8.8 Changes in Optimal Initial *SN* with Changes in Cost Ratio and CV of *SN*

8.5 Maintenance Warranties

Since maintenance warranties represent only a special case of long-term warranties, the formulation of the risk cost and warranty-based optimization models is similar. In fact, the models for maintenance warranties differ in only two main features; first, the limit state function needs to account for the accumulated traffic applications at the beginning of a warranty contract, and second, the optimization model needs to consider only the optimal maintenance schedule as decision variables, not the initial level of *SN*.

Given that the stress is defined by Equation 8.4, the strength by Equation 8.1, and that A represents the estimated accumulated load applications at the beginning of the warranty contract, the joint limit state function can be expressed as:

$$G(SN, M_r, ESAL_0, r, t, A) = \log W_{18} - \log N(t) - A \tag{8.18}$$

By evaluating the limit state function, using the method of moments, the conditional reliability function of a pavement can be developed as presented in Chapter 5.

$$R(SN, M_r, ESAL_0, r, t \,|\, A) = \frac{R(SN, M_r, ESAL_0, r, t, A)}{R(SN, M_r, A)} \tag{8.19}$$

If the estimate of A, the accumulated load applications at the beginning of the maintenance warranty contract, is not available, one can use the available measurements, such as deflection measurements, to calculate the effective *SN*, and subsequently to estimate A. To be consistent with the AASHTO design process (AASHTO, 1993), let the remaining life of a pavement after A accumulated load applications be defined as:

$$R_L = \frac{W_{18} - A}{W_{18}} \tag{8.20}$$

Then, based on the calculation of the effective *SN*, one can obtain the value of the effective remaining life, and subsequently the value of the accumulated load applications A. With the estimated *A*, and the effective *SN* can be calculated as:

$$SN_{eff}(t) = SN\left[1 - 0.7 \times \exp\left[-\left(\frac{W_{18} - A - N(t)}{W_{18}} + 0.85\right)^2\right]\right] + \Delta SN \tag{8.21}$$

Finally, the objective function of the warranty-based maintenance scheduling optimization model that considers *j* rehabilitations can be defined as:

$$\min_{SN, \Delta SN^j, \mathbf{t}_{\Delta SN^j}} C(SN) + C(\Delta SN^j) + ERC(\cdot, SN, \Delta SN^j, \mathbf{t}_{\Delta SN^j}, W) \tag{8.22}$$

8.5.1 Numerical Example

The considered section for maintenance warranties is also a 5-mile-long asphalt concrete pavement section with four lanes. The pavement was constructed with the initial *SN* value of 3.61. Using the direct method, it is estimated that 4 million ESALs have been applied to the pavement, without causing the pavement to fail. The objective of the analysis is to determine the expected risk cost and the optimal maintenance strategy for the warranty period of 10 years.

Without accounting for preventive maintenance, the total expected costs consist of two components, the rehabilitation cost and the expected risk cost. The relationship between them can be interpreted as: investing in rehabilitation to reduce the risk of failure vs. postponing the investment in rehabilitation and taking the higher risks of failure. Naturally, the key factor in the analysis is the consequence of a failure, or the failure cost. If the failure cost is not the dominant cost component, the optimal solution would be to avoid rehabilitations until the pavement fails, and then apply the emergency repair action.

Figure 8.9 shows the comparison of two different scenarios: apply rehabilitation vs. avoid any rehabilitation. It can be observed from Figure 8.9 that if the ratio of the failure and rehabilitation costs is smaller than 2.7, it is more cost-effective to skip the rehabilitation. In this case, the expected risk cost does not justify the spending on rehabilitation, and the contractor can take a "wait-and-see" position. However, if the warranty specifies the failure cost that causes the ratio to exceed the value of 2.7, then the application of rehabilitation would be on average a less costly option.

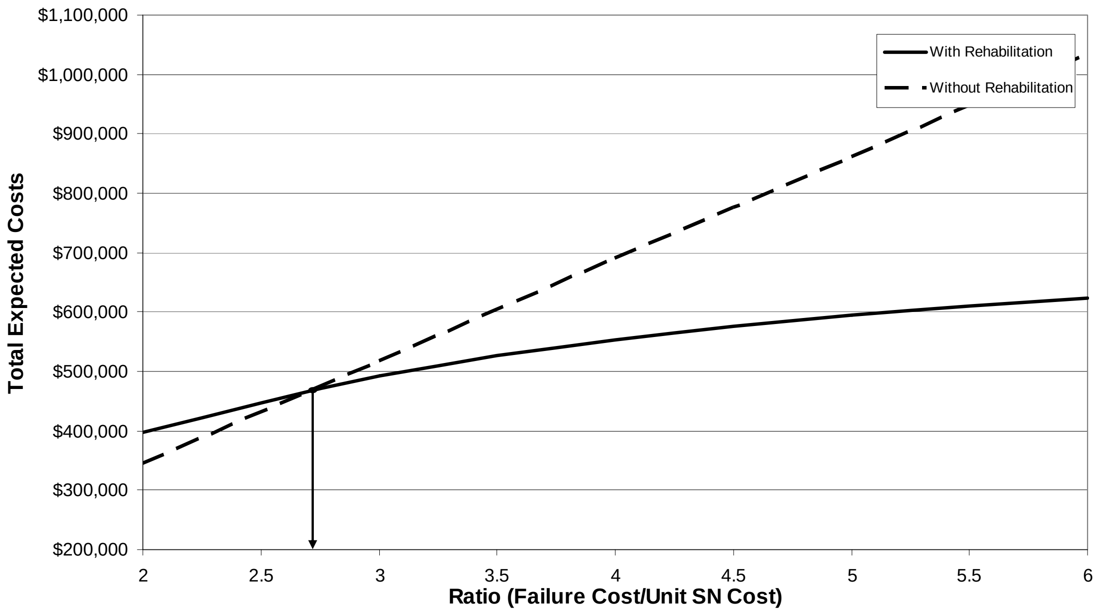


Figure 8.9 Effect of the Failure to Unit *SN* Cost Ratio on Total Expected Costs

Assuming that the considered warranty contract specifies the failure cost of $75,000 per lane per mile, or three times that of the unit cost of *SN*, five different scenarios are considered. In scenario I, no preventive maintenance or rehabilitation is taken; only corrective repairs are conducted when the pavement fails. Scenario II and Scenario III consist of applying an overlay that will increase the effective *SN* by 0.8 at year 4 and an overlay that will increase the effective *SN* by 0.9 at year 5, respectively, while Scenario IV and V include applying a preventive maintenance action at year 4 and 5, respectively. Table 8.6 summarizes the expected risk costs associated with these scenarios and presents the estimates for particular scenarios.

Table 8.6 Scenarios Considered in Numerical Example

Scenario	Time of Applying PM&R Action	Type of PM&R Action	Expected Risk Cost	Value of PM&R Action
1	-	-	$516,900	-
2	4	Rehab, dSN=0.8	$220,600	$296,300
3	5	Rehab, dSN=0.9	$234,000	$282,900
4	4	PM	$415,800	$101,100
5	5	PM	$411,600	$105,300

It can be observed from Table 8.6 that the application of preventive maintenance and rehabilitations (PM&R) can significantly reduce the risk cost. The value of these actions range from approximately $100,000 for preventive maintenance actions to almost $300,000 for the rehabilitation strategy defined in Scenario II. In general, the results indicate that a "reasonable" value of such maintenance contracts is approximately between $250,000 and $500,000.

Compared with the benchmark Scenario I, the most cost-effective scenario is Scenario II; applying the rehabilitation that will increase the effective *SN* by 0.8 at year 4 from the beginning of the contract would yield the total expected risk cost of approximately $220,600. The remaining life of a pavement with the application of such a rehabilitation strategy is estimated to be 0.11. This low value of the remaining life indicates that for the considered level of the failure to unit *SN* cost ratio, the decision-maker should still take risks until the risk cost component becomes dominant. In fact, as the ratio increases, the decision-maker should consider the rehabilitations that will reduce the risk cost sooner rather than later. This is illustrated in Figure 8.10. As the ratio increases, the optimal time to rehabilitation decreases. On the other hand, an increase in the ratio yields an increase in the optimal ΔSN^*; in other words, it is more cost-effective to apply a thicker overlay.

To estimate the effects of the reduced variability of the pavement strength, as well as the effects of more accurate prediction of the future traffic loading, a sensitivity analysis is conducted. A dual variable that corresponds to the coefficient of variation of *SN* indicates that a 0.01 decrease in CV[*SN*] causes a reduction in the total expected costs by $11,570; similarly, a 0.01 decrease in CV[$ESAL_0$] would reduce the total expected costs by $200. Since the variability of the pavement strength increases with the length of the sections, special attention should be given to the problem of defining a "suitable" length of the sections considered for maintenance warranty contracts.

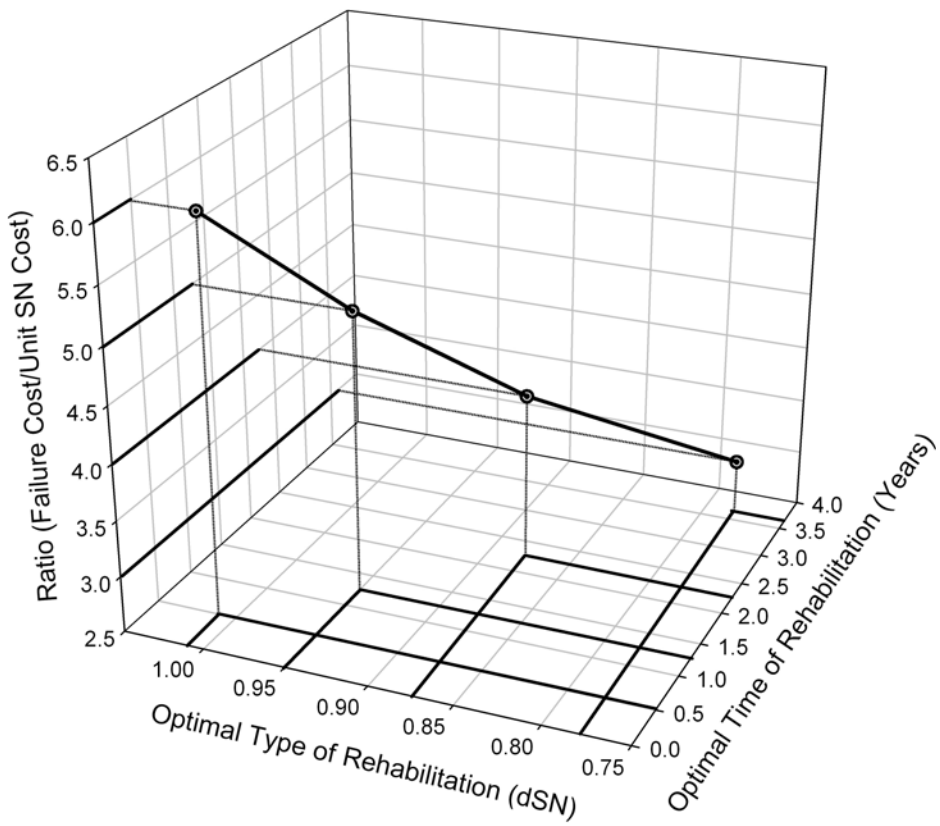


Figure 8.10 Optimal Rehabilitation Schedule and the Ratio (Failure Cost/Unit *SN* Cost)

8.6 Summary

This chapter presents a case study to test the accuracy of the method of moments and to illustrate the overall methodology. The first section presents the current AASHTO method for design of pavements, while the second section presents a comparison study between the method of moments and Monte Carlo simulation. The third, fourth, and fifth section of this chapter illustrate the risk cost quantification and the warranty-based optimization models for short-term, long-term, and maintenance warranties. In the following chapter, a summary of the research findings is presented.

CHAPTER 9 SUMMARY OF MAJOR FINDINGS AND RECOMMENDATIONS FOR FUTURE RESEARCH

This chapter summarizes major findings, discusses limitations, and suggests directions for future research. The chapter is organized in two sections. In the first section, a summary of the dissertation work is presented, with special emphasis on positioning the problem of performance warranties in the context of the existing knowledge in transportation infrastructure. In the second section, the limitations of the developed methodology are presented, and the directions for future work are suggested. The discussion includes possible extensions of the methodology as well as the identification of research needs in this new area of transportation infrastructure engineering.

9.1 Summary

The overall goal of this research is to develop a comprehensive methodological framework to model life-cycle costs associated with performance warranties for transportation infrastructure. Since performance warranties are a relatively recent development in the transportation industry, this dissertation research addresses a fundamental problem related to performance warranties: quantifying the performance-related risk costs. Clearly, this research subject requires knowledge in transportation infrastructure engineering, economics theory, and construction management. More specifically, this dissertation research is about developing the models to support the evaluation of performance warranties from the perspectives of all parties involved. From the perspective of transportation agencies, the objective is to find an upper bound on the risk cost that the contractors would be allowed to include in their bidding price; on the other hand, from the perspective of the contractors, the objective is to determine a design-build strategy that minimizes the total project cost.

This dissertation work focuses on three major topics. The first topic is the formulation of a framework for characterizing performance warranties. The discussion of this topic can be found in Chapter 3, where an integrated framework for the study of warranties is presented, and the relationships among significant factors influencing the warranty system are identified and discussed. A descriptive representation of warranty systems is developed for three categories of transportation infrastructure warranties, namely, short-term, long-term, and maintenance warranties.

The second topic is the development of probabilistic performance models that consider two important problems: first, how to obtain a closed-form solution for a more accurate estimation of the failure probabilities, and second; how to model the effects of rehabilitations on the rate of occurrence of failures.

To obtain more accurate estimates of failure probabilities, and at the same time keep the solution analytically tractable, the developed reliability model is based on the method of moments, a technique that utilizes the information of the first four central moments in estimating cumulative failure probabilities. The benefits of using the method of moments with the explicitly defined limit state functions are: 1) the consideration of different sources of uncertainty associated with transportation infrastructure, both in utilization and structural behavior, 2) a high accuracy of the cumulative failure probability estimation; and 3) a closed-form solution for estimating reliability functions. Furthermore, this method for developing reliability functions is not limited to linear limit state formulations with normal basic random variables, but it can also be applied to highly nonlinear limit state functions, where the basic random variables can take the form of any distribution. The developed classic reliability model is then altered to include the assessment of conditional reliability and the effects of preventive maintenance actions. These reliability-based models are presented in Chapter 4.

To model the effects of repairs and rehabilitations on the deterioration process, probabilistic performance models that take into account the effects of emergency repairs and rehabilitations are developed and presented in Chapter 5. These models are based on the non-homogeneous Poisson process assumption, where the rehabilitation effects are considered explicitly through their impact on the design variable in the limit state function. More specifically, the effects of rehabilitation are modeled directly through the rate of occurrence of failures function. The main advantage of this approach is the establishment of a clear connection between the reliability model, discussed in Chapter 4, and the stochastic counting models, discussed in Chapter 5. In the subsequent case study, presented in Chapter 8, the limit state function for both the reliability models and the performance models based on the Non-homogeneous Poisson process is developed using the current 1993 AASHTO design equations for flexible pavements.

Finally, the third topic of this dissertation is the formulation and analysis of the risk cost quantification and warranty-based optimization models for determining the optimal design strategy and maintenance schedule. Because of their flexibility and robustness, the risk cost quantification models can be applied to different warranty specifications and different types of transportation infrastructure facilities. The warranty analysis is further expanded to include the formulation of warranty-based optimization models. These optimization models are different from the traditional models used in transportation infrastructure management, mainly for the fact that they consider the risk cost as a cost component, in contrast to the traditional ones where the risk cost is not explicitly considered. The risk cost quantification models and the warranty-based optimization models are presented in Chapter 6 and Chapter 7, respectively.

9.2 Directions for Future Research

Although this dissertation work has produced a methodology for a comprehensive study of performance warranties, it can, by no means, solve all the problems associated with warranty-based contracting. In fact, many important problems still remain open, and ultimately, need to be resolved if warranty-based contracting is to become a more prevalent contracting method for the procurement and management of transportation infrastructure. Some of the identified problems requiring further research attention include:

1. The development of probabilistic performance models that can predict the failure probabilities and the expected number of failures for multiple performance indicators: this problem is important for warranty contracts that specify multiple failure criteria for different distress indicators, such as roughness and fatigue cracking, among others. Since the scope of this research was limited to a single

failure criterion, the performance models developed in this dissertation do not consider multiple performance indicators.

2. The development of reliability models that can distinguish among the effects of different preventive maintenance actions: even though the developed performance models are generally capable of differentiating between the effects of preventive maintenance and rehabilitations, still they are not capable of distinguishing among the effects of different preventive maintenance actions. For example, seal coats and fog seals both abate the pavement deterioration process; however, their effects on the deterioration process are quite different. This is something that is not captured by the models developed in this dissertation. Modifying the proposed reliability model to include a calibration for the effects of preventive maintenance would provide a better insight to the cost-effectiveness of different preventive maintenance options under the coverage of warranties.
3. The validation of the risk cost estimates in practice: this is of great importance to the successful implementation of the proposed integrated framework, so that it can serve as a solid foundation for the quantitative analysis of performance warranties. In spite of the abundance of data for modeling the warranty servicing cost of manufactured products, data for validating the estimated risk cost of transportation infrastructure are not easily obtainable. Part of the reason is that such data are often considered proprietary, as it directly affects the contractors' position in a bidding process. To overcome this difficulty, a closer collaboration

among the contracting industry, transportation agencies, and researchers is highly desirable.

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